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ON THE STRUCTURE OF PRODUCED AND INDUCED INDECOMPOSABLE LIE MODULES

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1. *Introduction.* In recent years, it has become evident that certain indecomposable modules are of basic importance to the representation theory of semisimple Lie algebras both over fields of characteristic 0 and over modular fields. If $\mathfrak{g}$ is a semisimple Lie algebra and $\mathfrak{b}$ a Borel subalgebra of $\mathfrak{g}$, then these fundamental $\mathfrak{g}$-modules are those induced from one-dimensional $\mathfrak{b}$-modules (cf. [12],[1],[5],[4]). Also modules induced from more general parabolic subalgebras have recently revealed some deep, far-reaching properties in the characteristic 0 setting (Lepowsky [7]). On the other hand, Humphreys' work [4] indicates that a better understanding of induced indecomposable modules would be desirable in the modular case too.

In this paper we study (restricted) indecomposable modules produced and induced from parabolic subalgebras of classical Lie algebras over fields of prime characteristic. We emphasize that ,,produced'' is being used here in the sense of [8],[13],[14]. Section 2 summarizes the basic definitions and properties of parabolic subalgebras and produced modules as presented in [8]. These are then applied in showing that indecomposable modules behave well under production: a produced module is indecomposable if and only if the original is so. The duality relation between induced and produced modules is then used in Section 4 to prove results about induced modules. In particular, if we start from an indecomposable module with a unique maximal submodule, then the resulting induced module is also indecomposable. A special case of this is the well-known fact that the standard cyclic $\mathfrak{g}$-modules Z_λ are indecomposable. As another corollary we see that modules induced from the principal indecomposable modules are all indecomposable.

2. *Preliminary results.* Let $\mathfrak{g}_c$ be a finite dimensional complex simple Lie algebra, and let Λ, Λ^+, and $\Delta = \{\alpha_1, ..., \alpha_l\}$, respectively, denote

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the sets of all roots, positive roots and simple roots of $\mathfrak{g}_c$ relative to a Cartan subalgebra $\mathfrak{h}_c$ (in some ordering). We fix a Chevalley basis $\{ e_1, \ldots, e_m ; h_1, \ldots, h_l ; f_1, \ldots, f_m \}$ of $\mathfrak{g}_c$, where $\{ h_1, \ldots, h_l \}$ is a basis for $\mathfrak{h}_c$ and e_i's (f_i's) are positive (resp. negative) root vectors. Let $\mathfrak{g}_z$ denote the free **Z**-module on this basis, and let F be a field of characteristic $p > 3$. Then $\mathfrak{g} = \mathfrak{g}_z \otimes_z F$ becomes a Lie algebra over F with a basis $\{ e_i \otimes 1 , h_k \otimes 1 , f_i \otimes 1 ; i = 1, \ldots, m , k = 1, \ldots, l \}$, and we denote these basis elements again simply by e_i, h_k, f_i. The algebras $\mathfrak{g}$ are called classical Lie algebras over the field F ([10]).

Let $\mathfrak{g}_\alpha$ be the root space corresponding to a root α, and let $\mathfrak{h}$ be the Cartan subalgebra of $\mathfrak{g}$ araising from $\mathfrak{h}_c$. A maximal solvable subalgebra $\mathfrak{b}$ of $\mathfrak{g}$ of the form $\mathfrak{b} = \mathfrak{h} + \sum_{\alpha \in \Delta^+} \mathfrak{g}_\alpha$ is given the name Borel subalgebra. We call any subalgebra $\mathfrak{p}$ of $\mathfrak{g}$ containing $\mathfrak{b}$ a *parabolic* subalgebra. Clearly $\mathfrak{p}$ is of the form $\mathfrak{p} = \mathfrak{p}_\pi = \mathfrak{h} + \sum_{\alpha \in \pi} \mathfrak{g}_\alpha$, where $\Delta^+ \subset \pi \subset \Delta$ and π satisfies the conditions i) $\Delta = \pi \cup (-\pi)$ and ii) π is closed under addition, i.e. $\alpha , \beta \in \pi$, $\alpha + \beta \in \Delta$ implies $\alpha + \beta \in \pi$.

Let $\pi_s = \pi \cap (-\pi)$, and let π_n be the complement of π_s in π. Define $\mathfrak{l}_\pi = \mathfrak{h} + \sum_{\alpha \in \pi_s} \mathfrak{g}_\alpha$, called the Levi subalgebra of $\mathfrak{p}_\pi$, $\mathfrak{n}_\pi^+ = \sum_{\alpha \in \pi_n} \mathfrak{g}_\alpha$ and $\mathfrak{n}_\pi^- = \sum_{\alpha \in \Delta - \pi} \mathfrak{g}_\alpha$. When no confusion is possible we usually drop the subindex π in this notation, and hence can write the classical Lie algebra $\mathfrak{g}$ in a form

$$\mathfrak{g} = \mathfrak{p} \oplus \mathfrak{n}^- = \mathfrak{n}^+ \oplus \mathfrak{l} \oplus \mathfrak{n}^- .$$

It is now easy to check that $\mathfrak{n}^+$ is an ideal of $\mathfrak{p}$, that $[\mathfrak{n}^+ , \mathfrak{l}] \subset \mathfrak{n}^+$, $[\mathfrak{n}^- , \mathfrak{l}] \subset \mathfrak{n}^-$ and that $\mathfrak{n}^+ , \mathfrak{n}^-$ are both nilpotent Lie algebras. Moreover, every classical Lie algebra is a restricted algebra ([10]) in the sense of [6]. Evidently $\mathfrak{n}^+$, $\mathfrak{n}^-$, $\mathfrak{l}$ and $\mathfrak{p}$ are all restricted subalgebras of $\mathfrak{g}$.

Let $\mathscr{U}(\mathfrak{g})$, $\mathscr{U}(\mathfrak{p})$, $\mathscr{U}(\mathfrak{n}^-)$ denote the u-algebras (=restricted universal enveloping algebras) of the corresponding restricted Lie algebras $\mathfrak{g}$, $\mathfrak{p}$, $\mathfrak{n}^-$. The restricted analogue of the Poincaré-Birkhoff-Witt theorem then gives

$$\mathscr{U}(\mathfrak{g}) = \mathscr{U}(\mathfrak{p}) \oplus \mathscr{U}(\mathfrak{p})\, \mathfrak{n}^- \mathscr{U}(\mathfrak{n}^-) .$$

Let $\gamma : \mathscr{U}(\mathfrak{g}) \to \mathscr{U}(\mathfrak{p})$ be the projection onto the first factor.

If W is a restricted $\mathfrak{l}$-module, then this can be extended to a restricted $\mathfrak{p}$-module $\widetilde{W}$ by setting $\mathfrak{n}^+ \widetilde{W} = 0$. Hence we can define a $\mathscr{U}(\mathfrak{g})$-module

$$P(W) = \mathrm{Hom}_{\mathscr{U}(\mathfrak{p})}(\mathscr{U}(\mathfrak{g}) , \widetilde{W})$$

by letting $(y f)(x) = f(x y)$ for all $f \in P(W)$, $x , y \in \mathscr{U}(\mathfrak{g})$. As an $\mathfrak{l}$-module we may injectively embed W into $P(W)$ via the map

$\omega : W \to P(W)$, which is defined by $\omega(v)(x) = \gamma(x)\, v$, $v \in W$, $x \in \mathcal{U}(\mathfrak{g})$. The following definition and the basic properties of produced modules are essentially due to Wallach [13],[14].

Definition. *The* $\mathcal{U}(\mathfrak{g})$*-submodule of* $P(W)$ *generated by* $\omega(W)$ *is called a produced module (or a module produced from the* $\mathfrak{l}$*-module* W*). It is denoted by* $W^* = \mathcal{U}(\mathfrak{g})\,\omega(W)$.

If V is a $\mathfrak{g}$-module, let $V^{\mathfrak{n}^-} = \{ v \in V \mid x\, v = 0,\ \text{for all } x \in \mathfrak{n}^- \}$. The following facts were proved in [8], [14].

Proposition 2.1. *Let* $\mathfrak{g}$ *be a classical Lie algebra and* $V \neq 0$ *a restricted* $\mathfrak{g}$*-module. Then*

i) $V^{\mathfrak{n}^-} \neq 0$.

ii) V *is produced from an* $\mathfrak{l}$*-module if and only if* $V = \mathcal{U}(\mathfrak{g})\, V^{\mathfrak{n}^-}$ *and* $\mathfrak{n}^+ V \cap V^{\mathfrak{n}^-} = 0$, *and in this case it is produced from the* $\mathfrak{l}$*-module* $V^{\mathfrak{n}^-}$.

iii) If W_1, W_2 *are two* $\mathfrak{l}$*-modules, then* $(W_1 \oplus W_2)^* \cong W_1^* \oplus W_2^*$ *as* $\mathfrak{g}$*-modules.*

iv) *Every irreducible* $\mathfrak{g}$*-module* V *is produced from the irreducible* $\mathfrak{l}$*-module* $V^{\mathfrak{n}^-}$.

3. *Produced indecomposable modules.* Frequently, the produced Lie modules seem to behave better than the induced ones. In [8] we saw how a $\mathfrak{g}$-module is completely reducible if and only if it is produced from a completely reducible $\mathfrak{l}$-module. The next proposition shows that produced indecomposable modules exhibit similar behaviour. From now on all modules and algebras are automatically assumed to be restricted.

Proposition 3.1. *Let* $\mathfrak{g}$ *be a classical Lie algebra and* $\mathfrak{l}$ *a Levi component of a parabolic subalgebra of* $\mathfrak{g}$. *Let* W *be an* $\mathfrak{l}$*-module. Then the produced* $\mathfrak{g}$*-module* W^* *is indecomposable if and only if* W *is an indecomposable* $\mathfrak{l}$*-module.*

Proof. Assume we could decompose W^* into a direct sum of two $\mathfrak{g}$-modules, $W^* = V_1 \oplus V_2$. If $f \in V_1^{\mathfrak{n}^-} \subset W^*$, then $f(x) = f(\gamma(x)) = \gamma(x) f(1) = \omega(f(1))(x)$. Hence $V_1^{\mathfrak{n}^-} \subset \omega(W)$. Similarly $V_2^{\mathfrak{n}^-} \subset \omega(W)$. Since $V_1 \cap V_2 = 0$, we now have

$$V_1^{\mathfrak{n}^-} \oplus V_2^{\mathfrak{n}^-} \subset \omega(W),$$

and we proceed to show that in fact this is an equality. To do this let

$$\omega(v) \in \omega(W) \subset W^* = V_1 \oplus V_2 .$$

Write $\omega(v) = f_1 + f_2$, where $f_1 \in V_1$, $f_2 \in V_2$. It is easy to see that $\omega(W) \subset (W^*)^{\mathfrak{n}^-}$, hence $0 = n\,\omega(v) = n f_1 + n f_2$ for all $n \in \mathfrak{n}^-$. But since $V_1 \cap V_2 = 0$, this forces $n f_1 = 0$, $n f_2 = 0$ for all $n \in \mathfrak{n}^-$. Hence $\omega(v) = f_1 + f_2$ with $f_1 \in V_1^{\mathfrak{n}^-}$, $f_2 \in V_2^{\mathfrak{n}^-}$.

Now $\omega(W)$ as an $\mathfrak{l}$-module is isomorphic to the indecomposable module W and $\omega(W) = V_1^{\mathfrak{n}^-} \oplus V_2^{\mathfrak{n}^-}$. It follows that $V_1^{\mathfrak{n}^-}$ or $V_2^{\mathfrak{n}^-}$ is $= 0$. Consequently $V_1 = 0$ or $V_2 = 0$ (Proposition 2.1) and hence W^* is indecomposable.

Conversely, if W^* is indecomposable, then so is the $\mathfrak{l}$-module W. Indeed, if we had $W = W_1 \oplus W_2$ this would imply $W^* \cong W_1^* \oplus W_2^*$. Since $W_i^* \neq 0$ if $W_i \neq 0$, this would contradict the indecomposability of W^*.

Remark. Since the map $\omega : W \to P(W)$ is injective, we have $\dim W < \dim W^*$. If π consists of Δ^+ together with a single simple root α_k, then $\mathfrak{l}_\pi = \mathfrak{h} + Fe_k + Ff_k$. In [9] Pollack considered first the structure of indecomposable modules for the classical Lie algebra of type A_1, hence also for the algebra $\mathfrak{l}_\pi$ above. In particular, he constructed indecomposable A_1-modules of arbitrary high dimensions. Since the dimension does not decrease in producing, it follows from Proposition 3.1 that every classical Lie algebra has (restricted) indecomposable modules of arbitrary high dimension. This is a result of Pollack ([9], Theorem 6).

If V is any produced $\mathfrak{g}$-module, then the first and last member in a composition series of V are produced. It would be interesting to know whether (or when) all members in a composition series of V have to be produced modules.

4. *Induced indecomposable modules.* Let $\mathfrak{g}$ be a classical Lie algebra, $\mathfrak{l}$ a subalgebra of $\mathfrak{g}$ and W an indecomposable $\mathfrak{l}$-module. In this section we consider the question: when is the induced $\mathfrak{g}$-module $\mathscr{U}(\mathfrak{g}) \otimes_{\mathscr{U}(\mathfrak{l})} W$ indecomposable? The results below generalize those in [8], §4.

A $\mathfrak{g}$-module V is called completely indecomposable (cf. [11]) if every submodule of V (including V) is indecomposable. We see at once that a module is completely indecomposable if and only if it has a unique minimal submodule. All irreducible modules are, of course, completely indecomposable; moreover, since $\mathscr{U}(\mathfrak{g})$ is a Frobenius algebra, the PIM's (=principal indecomposable modules) of $\mathscr{U}(\mathfrak{g})$ are completely indecomposable (this follows from Theorem 58.12 in [3]). In addition the standard cyclic modules [4] have a unique maximal submodule, hence their contragredient modules are completely indecomposable.

Lemma 4.1. *Let $\mathfrak{l}$ be the Levi component of a parabolic subalgebra of $\mathfrak{g}$. Then a produced $\mathfrak{g}$-module W^* is completely indecomposable if and only if the $\mathfrak{l}$-module W is completely indecomposable.*

Proof. If W is not completely indecomposable it has a submodule of the form $W_1 \oplus W_2$, $W_i \neq 0$. But then $W_1^* \oplus W_2^* \cong (W_1 \oplus W_2)^* \subset W^*$ shows that neither can W^* be completely indecomposable.

On the other hand, if W is completely indecomposable, then so is W^*. Indeed, if $V_1 \oplus V_2 \subset W^*$ then, as in the proof of Proposition 3.1, $V_1^{\mathfrak{n}^-} \oplus V_2^{\mathfrak{n}^-} \subset \omega(W) \cong W$.

Our main tool in dealing with induced modules is the duality relation between induced and produced modules. This is made precise in the following lemma (for the proof see [2], Prop. 1).

L e m m a 4.2. *Let* $\mathfrak{g}$, $\mathfrak{p}$ *and* $\mathfrak{l}$ *be as before. If* V *is a module, denote by* V' *its contragredient module* $\mathrm{Hom}_F(V, F)$. *Then for any* $\mathfrak{p}$*-module* W *we have a (restricted)* $\mathfrak{g}$*-module isomorphism*

$$(\mathscr{U}(\mathfrak{g}) \otimes_{\mathscr{U}(\mathfrak{p})} W)' \cong P(W') = \mathrm{Hom}_{\mathscr{U}(\mathfrak{p})}(\mathscr{U}(\mathfrak{g}), W'),$$

sending any $\psi' \in (\mathscr{U}(\mathfrak{g}) \otimes W)'$ *onto* $\psi \in P(W')$, *which is defined by*

$$\psi(x)(v) = \psi'(x^s \otimes v), \quad x \in \mathscr{U}(\mathfrak{g}), \ v \in W.$$

Here s *is the Hopf algebra antipode of* $\mathscr{U}(\mathfrak{g})$ *(i.e. the unique antiautomorphism of* $\mathscr{U}(\mathfrak{g})$ *defined by* $x^s = -x$ *for all* $x \in \mathfrak{g}$*)*.

P r o p o s i t i o n 4.3. *If* W *is an indecomposable* $\mathfrak{l}$*-module with a unique maximal submodule, then the induced* $\mathfrak{g}$*-module* $\mathscr{U}(\mathfrak{g}) \otimes_{\mathscr{U}(\mathfrak{p})} W$ *is also indecomposable.*

Proof. We note that W is here first extended to a $\mathfrak{p}$-module by letting $\mathfrak{n}^- W = 0$. Since W has a unique maximal submodule, it follows that W' is completely indecomposable. Hence the $\mathfrak{g}$-module $(W')^*$ is also completely indecomposable.

Let $V \neq 0$ be a minimal $\mathfrak{g}$-submodule of $P(W')$. As before, it is easy to see that $0 \neq V\mathfrak{n}^- \subset \omega(W')$. But since V is minimal, we must have $V = \mathscr{U}(\mathfrak{g})\, V\mathfrak{n}^-$, hence

$$V \subset \mathscr{U}(\mathfrak{g})\, \omega(W') = (W')^*.$$

The module $(W')^*$, however, contains a unique minimal submodule, hence V is a unique minimal submodule of $P(W')$. Consequently $P(W') \cong (\mathscr{U}(\mathfrak{g}) \otimes W)'$ is (completely) indecomposable. Since a module is indecomposable if and only if its contragredient module is so, the proposition now follows.

Since every PIM has a unique maximal submodule ([3], 54.11), we get the following result.

C o r o l l a r y. *Let* P *be a* PIM *of* $\mathscr{U}(\mathfrak{l})$. *Then the* $\mathfrak{g}$*-module* $\mathscr{U}(\mathfrak{g}) \otimes P$ *is also indecomposable.*

We now continue on the assumption that W is an indecomposable $\mathfrak{l}$-module with a unique maximal submodule. It follows from the proof of Proposition 4.3 that the induced indecomposable $\mathfrak{g}$-module $\mathscr{U}(\mathfrak{g}) \otimes W$ has a unique maximal submodule. In the special case where W is an irreducible

$\mathfrak{l}$-module we can give a description of this maximal submodule using the theory of produced modules.

L e m m a 4.4. *Let* W *be an irreducible* $\mathfrak{l}$*-module. Then there exists a* $\mathfrak{g}$*-module isomorphism* φ *from* $(W^*)'$ *into* $(\mathscr{U}(\mathfrak{g}) \otimes_{\mathscr{U}(\mathfrak{p})} W)'$.

Proof. It follows from the basic properties of produced modules that $W^* \cong (W^*)^{\mathfrak{n}^+} \oplus \mathfrak{n}^- W^*$ as an $\mathfrak{l}$-module (cf. Prop. 2.1). Hence $(W^*)' \cong ((W^*)^{\mathfrak{n}^+})' \oplus (\mathfrak{n}^- W^*)'$. Let p_1 , p_2 denote the $\mathfrak{l}$-module projections given by this isomorphism. If $f \in ((W^*)')^{\mathfrak{n}^-}$ and $p_1(f) = 0$, then

$$f(v) = p_1(f)(v_1) + p_2(f)(v_2) = 0$$

for any $v = v_1 + v_2 \in W^* \cong (W^*)^{\mathfrak{n}^+} \oplus \mathfrak{n}^- W^*$. It follows that p_1 restricted to $((W^*)')^{\mathfrak{n}^-}$ gives an $\mathfrak{l}$-module isomorphism

$$((W^*)')^{\mathfrak{n}^-} \cong ((W^*)^{\mathfrak{n}^+})' \cong W' .$$

Since W^* is now an irreducible $\mathfrak{g}$-module we can combine Proposition 2.1 ii), iv) and Lemma 4.2 to get the following sequence of $\mathfrak{g}$-isomorphisms:

$$(W^*)' \cong (((W^*)')^{\mathfrak{n}^-})^* \subset P(((W^*)')^{\mathfrak{n}^-}) \cong P(W') \cong (\mathscr{U}(\mathfrak{g}) \otimes_{\mathscr{U}(\mathfrak{p})} W)' .$$

This proves the lemma.

P r o p o s i t i o n 4.5. *Let* W *be an irreducible* $\mathfrak{l}$*-module and* φ *the isomorphism established in the previous lemma. Then*

$$S = \{ v \in \mathscr{U}(\mathfrak{g}) \otimes W \mid \varphi(f)(v) = 0 \text{ for all } f \in (W^*)' \}$$

is the unique maximal submodule of the indecomposable module $\mathscr{U}(\mathfrak{g}) \otimes_{\mathscr{U}(\mathfrak{p})} W$.

Proof. Let N be a proper submodule of $\mathscr{U}(\mathfrak{g}) \otimes W$ properly containing S . Let $N^\perp$ consist of those $f \in (\mathscr{U}(\mathfrak{g}) \otimes W)'$ such that $f(N) = 0$. If $(W^*)' \cap N^\perp \neq 0$, then the irreducibility of $(W^*)'$ implies $(W^*)' \subset N^\perp$. But this means $N \subset S$, contrary to our choice of N .

Hence it is enough to find a nonzero $f \in (W^*)'$ such that $\varphi(f)(N) = 0$.

Let v_0 be a maximal weight vector in W , in which case $\omega(v_0)$ is a maximal weight vector in W^* . Since $(W^*)'$ is an irreducible $\mathfrak{g}$-module, we can represent it in a produced module form,

$$(W^*)' = \mathscr{U}(\mathfrak{g})\, \omega(((W^*)')^{\mathfrak{n}^-}) .$$

Choose $\psi \in ((W^*)')^{\mathfrak{n}^-}$ such that ψ vanishes on $\mathfrak{n}^- W^*$, but $\psi(\omega(v_0)) \neq 0$. Then $\varphi(\omega(\psi)) \in (\mathscr{U}(g) \otimes W)'$, and we proceed to a closer study of this element.

Now $\omega(\psi)$ is the map

$$\omega(\psi) : \mathscr{U}(\mathfrak{g}) \to ((W^*)')^{\mathfrak{n}^-} ,$$
$$\omega(\psi)(x) = \gamma(x)\, \psi ,$$

hence an element in $P(((W^*)')^{\mathfrak{n}^-})$. Identifying $((W^*)')^{\mathfrak{n}^-}$ with W' as in the proof of Lemma 4.4 we see that $\omega(\psi)$ can be considered as an element of $P(W')$. Hence using the isomorphism of Lemma 4.2, we see that $\varphi(\omega(\psi))$ is the map

$$\varphi(\omega(\psi)) : \mathcal{U}(\mathfrak{g}) \otimes W \to F ,$$

defined by

$$\varphi(\omega(\psi))(x \otimes v) = \omega(\psi)\,(x^s)\,(\omega(v)) .$$

This implies that

$$\varphi(\omega(\psi))(1 \otimes v_0) = \omega(\psi)(1)(v_0) = \gamma(1)\,\psi(\omega(v_0)) = \psi(\omega(v_0)) \neq 0 ,$$

whereas if $x \in \mathcal{U}(\mathfrak{g})$, $x \notin \mathcal{U}(\mathfrak{h})$, then

$$\begin{aligned}\varphi(\omega(\psi))(x \otimes v_0) &= \omega(\psi)(x^s)(\omega(v_0)) = \gamma(x^s)\,\psi(\omega(v_0)) \\ &= -\psi(\gamma(x^s)\,\omega(v_0)) = 0 .\end{aligned}$$

Now $1 \otimes v_0$ is a generator for $\mathcal{U}(\mathfrak{g}) \otimes W$ so that $1 \otimes v_0 \notin N$. Hence $\varphi(\omega(\psi)) \neq 0$ vanishes on N. As observed earlier, this proves the proposition.

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