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SEQUENTIAL CONVERGENCES AND DUNFORD–PETTIS PROPERTIES

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Abstract. Several forms of the Dunford–Pettis property are studied, each related to a different mode of sequential convergence, and a different class of weakly compact functions. The relationship between these Dunford–Pettis properties is investigated, and the appearance of previously studied Dunford–Pettis properties is pointed out, giving a unifying approach to the subject.

Introduction

In recent years several forms of the Dunford–Pettis property have been studied (see [6], [2], [12], [16]) which conform more or less to the following scheme: if τ is some type of sequential convergence in a Banach space X, and $\mathscr{A}$ is a class of functions (linear, polynomial, holomorphic) defined on X, one can define a Dunford–Pettis property on X by requiring

"For all Y, all weakly compact $F \in \mathscr{A}(X,Y)$, and all τ-null sequences (x_n) in X, $F(x_n)$ converges in norm to $F(0)$."

Clearly, the stronger the convergence τ is, the weaker the corresponding property will be; and larger classes of functions $\mathscr{A}$ will result in stronger properties. Our aim in this paper is to clarify the relationships between some of these properties and give a unified approach to the matter.

We find that—for any fixed type of convergence τ—taking $\mathscr{A}$ to be the class of linear operators, k-homogeneous polynomials, or holomorphic functions of bounded type produces the same Dunford–Pettis property. On the other hand, if one takes $\mathscr{A}$ to be the class of all holomorphic functions, a strictly stronger property may be obtained. We give conditions under which this happens. Finally, we concentrate on $\tau = H$ (holomorphic sequential convergence) and find that in this case even the strongest form of Dunford–Pettis property—when $\mathscr{A}$ is the class of all holomorphic functions—holds for any Banach space.

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Throughout, X will be a Banach space. If Y is another Banach space, $L(X,Y)$ will denote the space of continuous linear operators from X to Y. A continuous k-homogeneous polynomial $P\colon X \longrightarrow Y$ is given by $P(x) = A(x,\dots,x)$, where A is a continuous k-linear function. P determines a unique A if we require it to be symmetric. The space of all such polynomials will be denoted by $P^k(X,Y)$ (or simply $P^k(X)$ if Y is the scalar field) and is a Banach space when endowed with the norm

$$\|P\| = \sup\{\|P(x)\| : \|x\| \leq 1\}.$$

A function $f\colon X \longrightarrow Y$ is holomorphic if it can be locally expressed as a uniformly convergent power series. We will denote the space of all such functions by $H(X,Y)$, and by $H_b(X,Y)$ the subspace of those functions which are bounded on the bounded subsets of X (i.e., the holomorphic functions of bounded type). These spaces will be denoted by $H(X)$ and $H_b(X)$ in the scalar-valued case, respectively. $H_b(X)$ is a Fréchet space when endowed with the topology τ_b of uniform convergence over bounded subsets of X. Note that $P^k(X)$ with the norm defined above is a closed subspace of $H_b(X)$. We consider $H(X)$ with the Nachbin topology τ_ω (see [10, 3.14]). We will say that a mapping is weakly compact if it sends some neighborhood of the origin onto a relatively weakly compact set.

For more on polynomials and holomorphic functions see [10], and for an extensive survey on the Dunford–Pettis property see [8].

We will use several different forms of sequential convergence in X. Dudley [11] defines a sequential convergence τ in X as a relation "$\rightarrow_\tau$" between sequences and elements of X such that:

(i) If $x_n = x$ for all n, then $x_n \rightarrow_\tau x$.

(ii) If $x_n \rightarrow_\tau x$ and (x_{n_k}) is a subsequence of (x_n), then $x_{n_k} \rightarrow_\tau x$.

For our purposes, we add the following two conditions:

(iii) If $x_n \rightarrow_\tau 0$ then (x_n) is weakly null.

(iv) $x_n \rightarrow_\tau x$ if and only if $x_n - x \rightarrow_\tau 0$.

Many topologies and sequential convergences have appeared in the literature in accordance with our definition. Among them:

(1) Weak convergence $\tau = \omega$.

(2) The convergence τ_s introduced by Pełczyński [14]. A sequence (x_n) is said to be τ_s-null for $s \in [0,1)$ if there exists a constant $c > 0$ such that for any $k \in \mathbf{N}$ and any collection of distinct natural numbers $n_1,\dots,n_k$, $\|\theta_1 x_{n_1} + \dots + \theta_k x_{n_k}\| \leq ck^s$ whenever $|\theta_i| = 1$.

(3) Weak p-summability $\tau = \omega_p$. The sequence (x_n) is said to be ω_p-null if $\big(x'(x_n)\big) \in l^p$ for each $x' \in X'$.

(4) The convergence $\tau = P(^{\leq k}X)$ [12]. In this case (x_n) is considered τ-convergent to x if $P(x_n)$ converges to $P(x)$ for any continuous scalar-valued polynomial P of degree $\leq k$.

(5) Polynomial convergence. The sequence (x_n) is polynomially convergent to x if $P(x_n)$ converges to $P(x)$ for any continuous scalar-valued polynomial P.

Note that this is equivalent to $f(x_n)$ converging to $f(x)$ for any holomorphic scalar-valued function of bounded type f.

(6) Holomorphic convergence [15]. Here (x_n) is H-convergent to x when $f(x_n)$ converges to $f(x)$ for any holomorphic scalar-valued function f.

τ-Dunford–Pettis properties

We begin by proving that for any fixed τ, the classes $\mathscr{A}$ of linear, polynomial, or bounded type holomorphic functions give rise to the same Dunford–Pettis property. We will use the following lemma.

Lemma 1. *If for every τ-null sequence (x_n) in X and for every k and every weakly null sequence (P_n) in $P^k(X)$, $P_n(x_n) \to 0$, then all scalar-valued polynomials are τ-sequentially continuous. Consequently, the same holds for scalar-valued holomorphic functions of bounded type.*

Proof. We need only check the conclusion for homogeneous polynomials. This we do by induction on the degree k of a homogeneous polynomial Q. For $k = 1$ this is clear because τ-convergence implies weak convergence. Suppose then that the result holds for $(k-1)$-homogeneous polynomials, and consider a τ-null sequence (x_n) and a k-homogeneous scalar-valued polynomial Q. Define $T_Q\colon X \longrightarrow P^{k-1}(X)$ by $T_Q(x) = A(x, \cdot, \ldots, \cdot)$ where A is the symmetric k-linear form associated to Q. T_Q is a continuous linear operator, so for all $\gamma \in P^{k-1}(X)'$, $\gamma\big(T_Q(x_n)\big) \to 0$, thus $\big(T_Q(x_n)\big)$ is a weakly null sequence of $(k-1)$-homogeneous polynomials, and by our inductive hypothesis $Q(x_n) = T_Q(x_n)(x_n) \to 0$.

If $f \in H_b(X)$, its Taylor series converges uniformly over bounded sets such as (x_n). Thus, f is also τ-sequentially continuous. ▫

Theorem 2. *For any Banach space X, the following are equivalent.*

(a) *For all Y, all weakly compact $T \in L(X,Y)$, and all τ-null sequences (x_n) in X, $T(x_n) \to 0$ in norm.*

(a′) *For all weakly null sequences $(\gamma_n) \subset X'$ and all τ-null sequences (x_n) in X, $\gamma_n(x_n) \to 0$.*

(b) *For all Y, all weakly compact $P \in P^k(X,Y)$ $(k \geq 1)$, and all τ-null sequences (x_n) in X, $P(x_n) \to 0$ in norm.*

(b′) *For all weakly null sequences $(P_n) \subset P^k(X)$ $(k \geq 1)$, and all τ-null sequences (x_n) in X, $P_n(x_n) \to 0$.*

(c) *For all Y, all weakly compact $F \in H_b(X,Y)$, and all τ-null sequences (x_n) in X, $F(x_n) \to F(0)$ in norm.*

(c′) *For all weakly null sequences $(f_n) \subset H_b(X)$ and all τ-null sequences (x_n) in X, $f_n(x_n) \to 0$.*

Proof. We show first that (c) and (c′) are equivalent. The equivalences (a) $\Leftrightarrow$ (a′) and (b) $\Leftrightarrow$ (b′) are analogous—and perhaps less technical—so we will omit them.

(c) implies (c′): Let (f_n) and (x_n) be as in (c′), and define $F: X \longrightarrow c_0$ by $F(x) = \big(f_n(x)\big)$. Since (f_n) is weakly null and point evaluations are continuous linear forms over $H_b(X)$, F is well defined. Since (f_n) is weakly bounded, it is τ_b-bounded (both topologies produce the same dual space), and thus bounded for the compact-open topology. Hence (f_n) are uniformly bounded on compact subsets of X, and so F is locally bounded. Thus, to see that F is holomorphic, we only need to prove weak holomorphicity. Take then $a \in l^1$, and consider $(a \circ F)(x) = \sum_n a_n f_n(x)$. Since $a \circ F$ is a uniform limit (over bounded sets) of holomorphic functions of bounded type, $a \circ F \in H_b(X)$ for each a, and thus $F \in H_b(X, c_0)$. Now consider the transpose $F^t: l^1 \longrightarrow H_b(X)$ given by $F^t(a) = a \circ F$. By [17, 3.4] if we see that F^t is weakly compact, then F is also weakly compact. So take a in the unit ball of l^1 and note that $F^t(a) = a \circ F$ is in the closed absolutely convex hull of $\{f_n : n \geq 1\} \cup \{0\}$. Since this set is weakly compact in the metric space $H_b(X)$, so is its hull [18, IV.11.4]. Thus F^t, and F, are weakly compact. Since (x_n) is τ-null, $\lim_n F(x_n) = F(0)$ exists, and

$$|f_n(x_n)| \leq |f_n(x_n) - f_n(0)| + |f_n(0)| \leq \|F(x_n) - F(0)\| + |f_n(0)|$$

so $\lim_n f_n(x_n) = 0$, because $\lim_n f_n(0) = 0$.

(c′) implies (c): Let F and (x_n) be as in (c). We may suppose $F(0) = 0$, and prove that $\lim_n F(x_n) = 0$. If this were not the case, take $\varepsilon > 0$ and a subsequence (x_{n_k}) with $\|F(x_{n_k})\| > \varepsilon$ for all k. Consider $(y'_k) \subset Y'$ of norm one and such that $y'_k\big(F(x_{n_k})\big) = \|F(x_{n_k})\|$. Now consider the transpose of F, $F^t: Y' \longrightarrow H_b(X)$. This is a continuous linear operator, and is weakly compact because F is. Take then a subsequence (y'_{k_j}) with $F^t(y'_{k_j})$ converging weakly to $g \in H_b(X)$. Since (x_n) is τ-null and $F^t(y'_{k_j}) - g$ is weakly null, then $\big(F^t(y'_{k_j}) - g\big)(x_{n_{k_j}}) \to 0$. But by Lemma 1 (note that the τ_b topology when restricted to $P^k(X)$ coincides with the norm topology) $g(x_{n_{k_j}}) \to 0$, a contradiction. Thus $\lim_n F(x_n) = 0$.

Clearly (c) implies (b) and (b) implies (a). Thus we will be done if we show that (a) $\Rightarrow$ (b′) and (b) $\Rightarrow$ (c).

(a) implies (b′): We prove this by induction. The case of degree $k = 1$ is simply (a) $\Rightarrow$ (a′), so suppose the result true for $(k-1)$-homogeneous polynomials, and consider (P_n) and (x_n) as in (b′) above. Define a sequence of polynomials (Q_n) by setting $Q_n = A_n(x_n, \cdot, \ldots, \cdot)$ where A_n is the symmetric k-linear form associated to P_n. We need to show that this sequence is weakly null. Define $P: X \longrightarrow c_0$ by $P(x) = \big(P_n(x)\big)$. P is weakly compact, for its transpose $P^t: l^1 \longrightarrow P^k(X)$ is weakly compact ($P^t(e_n) = P_n \to 0$ weakly). Thus $P^{tt}: P^k(X)' \longrightarrow l^\infty$ has image contained in c_0. For each $L \in P^{k-1}(X)'$ define $\overline{L}: X \longrightarrow P^k(X)'$ by $\overline{L}(x)(R) = L\big(B(x, \cdot, \ldots, \cdot)\big)$, where B is the symmetric k-linear form associated to R. Now consider $P^{tt} \circ \overline{L}: X \longrightarrow c_0$. This is a weakly compact linear operator, and since (x_n) is τ-null, we have, by (a), that $P^{tt} \circ \overline{L}(x_n)$

tends to zero in norm, that is $\sup_j |(\overline{L}(x_n) \circ P^t)_j| \to 0$ as n grows. In particular

$$|L(Q_n)| = \big|L\big(A_n(x_n, \cdot, \dots, \cdot)\big)\big| = |\overline{L}(x_n)(P_n)| \to 0.$$

Thus (Q_n) is weakly null, and by our inductive hypothesis $P_n(x_n) = Q_n(x_n) \to 0$.

(b) implies (c): Since τ-null sequences are bounded, the Taylor series of f converges uniformly over (x_n). Also, each k-homogeneous polynomial in this series is weakly compact, so (b) applies. ▫

We are now in a position to define the τ-Dunford–Pettis property, given a sequential convergence τ.

Definition. We will say that a Banach space X has the τ-Dunford–Pettis property if one—and therefore all—of the conditions in Theorem 2 hold.

Note that the classical Dunford–Pettis property is obtained when τ is weak convergence. Other forms of the Dunford–Pettis property have been studied which conform to this scheme. Among them:

(1) $\tau = \omega_p$ produces the DPP_p studied by Castillo and Sánchez [6].

(2) $\tau = P(^{\leq k}X)$ corresponds to the polynomial Dunford–Pettis property studied by Farmer and Johnson [12].

(3) Polynomial convergence gives rise to the polynomial Dunford–Pettis property as defined by Biström, Jaramillo and Lindström [2].

Next, we prove that in the definition of the τ-Dunford–Pettis property, the class $\mathscr{A}$ of holomorphic functions of bounded type (or the space of k-homogeneous polynomials, or that of linear functions) cannot be replaced by the class of all holomorphic functions over X. Indeed, the resulting property is in general strictly stronger than the τ-Dunford–Pettis property. We will use the following definition.

Definition. We will say a Banach space X has the $*$-Dunford–Pettis property if for all weakly null sequences (x_n) in X and all weak-$*$ null sequences (x'_n) in X', $x'_n(x_n)$ converges to 0.

This property implies the Dunford–Pettis property. Note that all Schur spaces have the $*$-Dunford–Pettis property. Also, if X has this property, all its complemented subspaces have it. For a Grothendieck space this property is equivalent to the classical Dunford–Pettis property, thus for instance l^∞ has the $*$-Dunford–Pettis property [8], and so does H^∞ (see Bourgain [4], [5]). The space $l^1 \bigoplus l^\infty$ is an example of a space having the $*$-Dunford–Pettis property which is neither Schur nor Grothendieck. It is easily seen that X has the $*$-Dunford–Pettis property if and only if every weakly relatively compact subset of X is limited.

On the other hand, it is not difficult to check that $*$-Dunford–Pettis property coincides with the DP$*$ property introduced in [3], that is, the fact that weak$*$ and Mackey convergence coincide sequentially in X'. We refer to [3] for further information about this property and its connection with differentiability.

A typical example of a space having the Dunford–Pettis property, but lacking the $*$-Dunford–Pettis property is c_0. In such a space although a weakly compact holomorphic function F of bounded type will map weakly null sequences into sequences converging in norm to $F(0)$, the same will not be true of all weakly compact holomorphic functions.

Theorem 3. *If X has the Dunford–Pettis property but not the $*$-Dunford–Pettis property, there are scalar-valued holomorphic functions f on X which map a weakly null sequence into a sequence not norm convergent to $f(0)$.*

Proof. Take (x_k) weakly null in X and (x'_k) weak-$*$ null in X' such that $x'_k(x_k)$ does not converge to 0. Passing to a subsequence and multiplying the x_k's by suitable (bounded) scalars, we may suppose that $x'_k(x_k) = 1$ for all k. Now let $0 < \varepsilon < 1$. We will extract subsequences (x_{k_j}) and (x'_{k_j}) inductively. Set $k_1 = 1$, and having defined $k_1, \dots, k_{r-1}$, define k_r as follows: since (x'_k) is weak-$*$ null, choose $n_r > k_{r-1}$ and such that

$$|x'_k(x_{k_{r-1}})| < \frac{\varepsilon}{2+\varepsilon}, \qquad \text{if } k \ge n_r,$$

and since (x_k) is weakly null, take $m_r > k_{r-1}$ and such that

$$\sum_{j<r} |x'_{k_j}(x_k)|^j < \frac{\varepsilon}{2}, \qquad \text{if } k \ge m_r.$$

Define $k_r = \max\{n_r, m_r\}$. Now construct $f\colon X \longrightarrow \mathbf{C}$ as $f(x) = \sum_{j=1}^{\infty} x'_{k_j}(x)^j$. Note that since (x'_{k_j}) is weak-$*$ null and $0 < \limsup_j \|x'_{k_j}\| < \infty$ this is an entire function on X which is not of bounded type. Finally, for each $r \in \mathbf{N}$ we have

$$|f(x_{k_r}) - 1| \le \sum_{j<r} |x'_{k_j}(x_{k_r})|^j + \sum_{j>r} |x'_{k_j}(x_{k_r})|^j < \frac{\varepsilon}{2} + \sum_{j=1}^{\infty} \left(\frac{\varepsilon}{2+\varepsilon}\right)^j = \varepsilon,$$

bearing in mind that $k_r \ge m_r$ and that for $j > r$, $k_j \ge n_{r+1}$. Thus $|f(x_{k_r}) - 1| < \varepsilon < 1$, and $f(x_{k_r})$ cannot tend to $0 = f(0)$. □

We show next that we can find spaces X without the $*$-Dunford–Pettis property in the following two situations: if X does not contain a copy of l^1; and if X is not Schur, and the unit ball of X' is weak-$*$ sequentially compact.

Proposition 4. *If X has the $*$-Dunford–Pettis property, then X contains a copy of l^1.*

Proof. By the Josefson–Nissenzweig theorem [13] there is a weak-$*$ null sequence (y'_n) in the unit sphere of X'. Take a sequence (y_n) in X with $\|y_n\| \le 2$

and $y'_n(y_n) = 1$ for all n. If X contains no copy of l^1, we may use the Rosenthal–Dor theorem [9, p. 201] to extract from this last sequence a subsequence (which we still call (y_n)) which is weakly Cauchy. From this, we extract another subsequence inductively: set $0 < \varepsilon < 1$, and $n_1 = 1$; and having defined $n_1, \dots, n_{r-1}$, choose $n_r > n_{r-1}$ such that $|y'_{n_r}(y_{n_{r-1}})| < \varepsilon$. Since (y_n) is weakly Cauchy, $x_k = y_{n_k} - y_{n_{k-1}}$ is weakly null. Setting $x'_k = y'_{n_k}$ we have (x_k) weakly null and (x'_k) weak-$*$ null such that

$$|x'_k(x_k) - 1| = |y'_{n_k}(y_{n_{k-1}})| < \varepsilon < 1.$$

Thus $x'_k(x_k)$ cannot converge to zero. □

The following result is obtained in [3]. Our proof is different and we include it for the sake of completeness.

Proposition 5. *If X has the $*$-Dunford–Pettis property, and the unit ball of X' is weak-$*$ sequentially compact, then X is Schur.*

Proof. We follow [15]. If X were not Schur, take a weakly null sequence (y_n) contained in the unit sphere of X. From (y_n) we extract, by the Bessaga–Pełczyński selection theorem [9, p. 42], a basic sequence, which we still call (y_n), and consider the coordinate functionals $y'_n \colon S \longrightarrow \mathbf{C}$ where S is the closed linear span of the y_n's. These have norm not larger than twice the basis constant of (y_n). Extend each y'_n to all of X by Hahn–Banach, preserving the norms (we still call them y'_n). Since this sequence is bounded and the unit ball of X' is weak-$*$ sequentially compact, choose a weak-$*$ convergent subsequence (y'_{n_k}) and call y' its weak-$*$ limit. Now set $x'_k = y'_{n_k} - y'$ and $x_k = y_{n_k}$. The sequence (x_k) is weakly null, (x'_k) weak-$*$ null, and for each k we have $x'_k(x_k) = 1$. □

Note that if X is weakly compactly generated, the unit ball of X' is weak-$*$ sequentially compact [9, p. 228]. For further information about spaces with weak-$*$ sequentially compact dual balls, we refer to [9, Chapter 13] and references therein. A hereditarily Dunford–Pettis space lacking the $*$-Dunford–Pettis property will contain a copy of c_0 [7, Proposition 2], but we do not know if in this case one can find a complemented copy.

Finally, we consider what happens when the convergence τ is holomorphic convergence, that is: (x_n) is H-convergent to x if $f(x_n) \to f(x)$ for any $f \in H(X)$. In [15] Petunin and Savkin prove that holomorphic convergence implies norm convergence in spaces which are weakly compactly generated. However, there are spaces such as l^∞ where holomorphic convergence does not imply norm convergence [1]. In spite of this fact, as we see below, the "H-Dunford–Pettis property" holds in all Banach spaces. We consider the strongest form of this property, by taking $\mathscr{A}$ as the class of all holomorphic functions.

Theorem 6. *In any Banach space X, the following hold.*

(i) *For all Y, all weakly compact $F \in H(X,Y)$, and all H-null sequences (x_n), $F(x_n) \to F(0)$ in norm.*

(ii) *For all weakly null sequences $(f_n) \in H(X)$ and all H-null sequences (x_n), $f_n(x_n) \to 0$.*

Proof. (i): Let F and (x_n) be as above. By [17, 3.7], F factors through a reflexive Banach space G, that is, there is a holomorphic function $g: X \longrightarrow G$ and a continuous linear operator $T: G \longrightarrow Y$ such that $F = T \circ g$. Since g is holomorphic, the sequence $g(x_n)$ is H-null. But G is reflexive, so by [15] $g(x_n)$ converges in norm to $g(0)$. Thus $F(x_n) = T\big(g(x_n)\big)$ is norm convergent to $T\big(g(0)\big) = F(0)$.

We now prove that (i) implies (ii). The proof is similar to (c) implies (c′) above. We define $F: X \longrightarrow c_0$ by $F(x) = \big(f_n(x)\big)$. This function is holomorphic, and once again $F^t: l^1 \longrightarrow H(X)$ given by $F^t(a) = a \circ F$ is weakly compact, but in this case, the weak compactness of the closed absolutely convex hull of $\{f_n : n \in \mathbf{N}\} \cup \{0\}$ is more involved. Consider the Nachbin–Coeuré topology τ_δ on $H(X)$ (see [10, 3.16] for definition). Note that τ_δ is bornological and coincides with τ_ω on bounded sets of $H(X)$ (see [10, 3.19]). Thus, closed absolutely convex hulls in both topologies are the same. Now Krein's theorem [18, IV.11.4] assures that this set is weakly compact (since it is complete for τ_δ, which is the Mackey topology). □

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