Annales Academiæ Scientiarum Fennicæ
Mathematica
Volumen 31, 2006, 287–295

THE TRACE CLASS IS A Q-ALGEBRA

David Pérez-García

Universidad Rey Juan Carlos, Área de Matemática Aplicada, ESCET
Departamental II, ES-28933 Móstoles (Madrid), Spain; david.perez.garcia@urjc.es

Abstract. We use an old multilinear generalization of Grothendieck's inequality (for which we give a new proof and a non-commutative version) to show that the Schatten space S_p is a Q-algebra with the Schur product for $1 \le p \le 2$.

1. Introduction and notation

In [10], Grothendieck stated what he called "the fundamental theorem of the metric theory of tensor products", to be known later as Grothendieck's theorem or Grothendieck's inequality. In [14], Lindenstrauss and Pełczyński gave a detailed proof of this result and stated some of its consequences and equivalent formulations, making use of the theory of $\mathscr{L}_p$-spaces and using also the p-summing operators recently introduced in [18]. Since then, several equivalent formulations, extensions and different proofs have been obtained, together with innumerable applications. One of these extensions is the 'non-commutative' version given by Pisier (the interested reader can consult [13] or [22] for two recent applications). [8], [9] and [21] provide excellent expositions on this and related topics.

Grothendieck's theorem can be seen as a matrix inequality associated to certain bilinear operators. In the late 70's, several authors investigated multilinear extensions of Grothendieck's matrix inequality (see, for instance, [3], [5], [24] and the references therein). Here, in Theorem 2.2, we give an elementary proof of one of these extensions (other proofs can be seen in [5] or [24]). In Corollary 2.3 we apply it to obtain that the Schatten space S_p, with $1 \le p \le 2$ is a Q-algebra with the Schur product, answering (at least partially) an old problem that goes back to the work of Varopoulos in the 70's. We refer to [16] for the history of the problem and the solution in the case $2 \le p \le 4$. Let us just say here that the case $4 \le p \le \infty$ is still open. Motivated by this application, we give a non-commutative extension of Theorem 2.2 in the line of Pisier's result.

The notation and terminology used along the paper are standard in Banach space theory, as for instance in [8] or [9]. These books are also our main references for basic facts, definitions and unexplained notation. However, before going any further, we shall establish some terminology. All along this paper all the operators

---

2000 Mathematics Subject Classification: Primary 47H60; Secondary 46J99, 46L05, 47L25.

Partially supported by BMF 2001-1284.

are supposed to be continuous. Given X, Y Banach spaces, $\mathscr{L}(X,Y)$ will denote the Banach space of linear (and continuous) operators between them. X^* will be the dual of X and B_X its unit ball. For a finite sequence $(x_i)_{i=1}^m \subset X$ and $1 \le p < \infty$, we will write $\|(x_i)_{i=1}^m\|_p^\omega$ to denote

$$\sup\left\{ \left(\sum_{i=1}^m |x^*(x_i)|^p \right)^{1/p} : x^* \in B_{X^*} \right\}.$$

A linear operator u: $X \longrightarrow Y$ is said to be absolutely summing if there exists a constant $K > 0$ such that $\sum_{i=1}^m \|u(x_i)\| \le K \|(x_i)_{i=1}^m\|_1^\omega$ for each finite sequence $(x_i)_{i=1}^m \subset X$. $\pi_1(u)$ will be the least of such constants K. We will use the fact that u: $X \longrightarrow Y$ is absolutely summing if and only if there exists a constant $K > 0$ such that, for every $m \in \mathbf{N}$, $\| \mathrm{id}_{l_1^m} \otimes u \colon l_1^m \otimes_\varepsilon X \longrightarrow l_1^m \otimes_\pi Y\| \le K$, where ε and π denote the injective and projective tensor norms respectively [8, Proposition 11.1]. If so, $\pi_1(u)$ is again the least of such constants. Grothendieck's theorem tells us that there exists a constant $K > 0$ such that for every $m \in \mathbf{N}$ and for every u: $l_1^m \longrightarrow l_2^m$, we have that $\pi_1(u) \le K\|u\|$. The Grothendieck constant K_G is the least of them.

More generally, given $s, r_1, \ldots, r_n \in [1, \infty)$ such that

$$\frac{1}{s} \le \frac{1}{r_1} + \cdots + \frac{1}{r_n},$$

we say that a multilinear operator T: $X_1 \times \cdots \times X_n \longrightarrow Y$ is $(s; r_1, \ldots, r_n)$-summing if there exists a constant $K > 0$ such that

$$\left(\sum_{i=1}^m \|T(x_i^1, \ldots, x_i^n)\|^s \right)^{1/s} \le K \prod_{j=1}^n \|(x_i^j)_{i=1}^m\|_{r_j}^\omega$$

for every choice of sequences $(x_i^j)_{i=1}^m \subset X_j$. Again $\|T\|_{(s;r_1,\ldots,r_n)}$ will denote the least of such constants.

Let $\lambda > 1$. A Banach space X is said to be an $\mathscr{L}_{\infty,\lambda}$-space if, for every finite-dimensional subspace $E \subset X$ there exists another finite-dimensional subspace F, with $E \subset F \subset X$ and such that there exists an isomorphism v: $F \longrightarrow l_\infty^{\dim F}$ with $\|v\| \, \|v^{-1}\| < \lambda$. As basic examples we have that, for every compact Hausdorff space K and for every measure space (Ω, Σ, μ), $C(K)$ and $L_\infty(\mu)$ are $\mathscr{L}_{\infty,\lambda}$-spaces for every $\lambda > 1$.

A Banach algebra is said to be a uniform algebra if it is isometrically isomorphic (as Banach algebra) to a closed subalgebra of $C(K)$ for some compact Hausdorff space K. By a Q-algebra we mean a commutative Banach algebra that is isomorphic (as Banach algebra) to C/I, where C is a uniform algebra and I a closed ideal of C. It is well known since the 60's that every Q-algebra is an operator algebra, that is, isomorphic to a closed subalgebra of $\mathscr{L}(H,H)$ for a Hilbert space H.

S_∞ will be the Banach space of all compact operators on l_2 (with the operator norm). The Schatten space S_p, with $1 \le p < \infty$, will be the Banach space of all compact operators on l_2 such that $\operatorname{tr}|u|^p < \infty$, equipped with the norm $\|u\|_{S_p} = (\operatorname{tr}|u|^p)^{1/p}$.

Finally, we have to define the Schur product in S_p. Using the canonical basis, any compact operator on l_2 can be viewed as an infinite matrix $(a_{k,h})_{k,h\ge 1}$. The Schur product $*$ of two such matrices is just the pointwise product, that is

$$(a_{k,h})_{k,h\ge 1} * (b_{k,h})_{k,h\ge 1} = (a_{k,h} b_{k,h})_{k,h\ge 1}.$$

2. The results

We will need the following

Lemma 2.1. *Let $n, m \in \mathbf{N}$ and let $u_i \in \mathscr{L}(l_1^m, l_2^m)$ with $\|u_i\| \le 1$, $1 \le i \le n$. We write $i\colon l_1^m \hookrightarrow l_2^m$ for the formal inclusion and we write $P_n\colon l_2^m \times \overset{n}{\cdots} \times l_2^m \longrightarrow l_1^m$ for the product operator $P_n\big((x_i^1)_{i=1}^m, \dots, (x_i^n)_{i=1}^m\big) = (x_i^1 \cdots x_i^n)_{i=1}^m$ (and also for its associate linear operator $P_n\colon l_2^m \otimes_\pi \cdots \otimes_\pi l_2^m \longrightarrow l_1^m$). If we define v_n with the following diagram*

$$\begin{tikzcd} l_1^m \otimes_\varepsilon \cdots \otimes_\varepsilon l_1^m \arrow[r, "v_n"] \arrow[d, "u_1\otimes\cdots\otimes u_n"'] & l_2^m \\ l_2^m \otimes_\pi \cdots \otimes_\pi l_2^m \arrow[r, "P_n"'] & l_1^m \arrow[u, "i"'] \end{tikzcd}$$

Then $\pi_1(v_n) \le K_G^n$.

Proof. We are going to prove it by induction. The case $n = 1$ is obvious. For the general case, it is easy to see that the following diagram commutes

$$\begin{tikzcd} (l_1^m \otimes_\varepsilon \cdots \otimes_\varepsilon l_1^m) \otimes_\varepsilon l_1^m \arrow[r, "v_{n+1}"] \arrow[d, "v_n\otimes \mathrm{id}"'] & l_2^m \\ l_2^m \otimes_\pi l_1^m \arrow[d, "\mathrm{id}\otimes u_{n+1}"'] & \\ l_2^m \otimes_\pi l_2^m \arrow[r, "P_2"'] & l_1^m \arrow[uu, "i"'] \end{tikzcd}$$

By the induction hypothesis, $\pi_1(v_n) \le K_G^n$. Therefore $\|v_n \otimes \mathrm{id}\| \le K_G^n$ and, thanks to Grothendieck's theorem, we obtain that

$$\pi_1(v_{n+1}) \le \|v_n \otimes \mathrm{id}\|\, \|\mathrm{id} \otimes u_{n+1}\|\, \|P_2\| \pi_1(i) \le K_G^{n+1}. \ \square$$

Now, we can give our elementary proof of the following multilinear Grothendieck's inequality.

Theorem 2.2. *For every* $m \in \mathbf{N}$, $n \geq 2$, $(a_{i_1\cdots i_n})_{i_j=1}^m \subset \mathbf{K}$ *and* $x_{i_1}^1, \ldots, x_{i_n}^n \in B_{l_2^m}$ *we have*

$$\left|\sum_{i_j=1}^m a_{i_1\cdots i_n} \sum_{k=1}^m x_{i_1}^1(k)\cdots x_{i_n}^n(k)\right| \leq K_G^{n-1} \sup_{|t_{i_j}|\leq 1}\left|\sum_{i_j=1}^m a_{i_1\cdots i_n} t_{i_1}\cdots t_{i_n}\right|.$$

Proof. Let us write $z_{i_n} := \sum_{i_1,\ldots,i_{n-1}=1}^m a_{i_1,\ldots,i_n} e_{i_1}\otimes\cdots\otimes e_{i_{n-1}} \in l_1^m \otimes_\varepsilon \overset{n-1}{\cdots} \otimes_\varepsilon l_1^m$. It is easy to see that:

$$\|(z_{i_n})_{i_n=1}^m\|_1^\omega = \sup\left\{\left|\sum_{i_j=1}^m a_{i_1\cdots i_n} t_{i_1}\cdots t_{i_n}\right| : |t_{i_j}| \leq 1\right\}.$$

For every $j \in \{1,\ldots,n-1\}$ we define the operator $u_j\colon l_1^m \longrightarrow l_2^m$, by $u_j(e_{i_j}) = x_{i_j}^j$, $(1 \leq i_j \leq m)$. Since (e_{i_j}) are the extremal points of the ball of l_1^m and since $\|x_{i_j}^j\| \leq 1$, we have that $\|u_j\| \leq 1$. Applying Lemma 2.1 we get

$$\sum_{i_n=1}^m \|v_{n-1}(z_{i_n})\| \leq K_G^{n-1}\|(z_{i_n})_{i_n=1}^m\|_1^\omega.$$

Finally,

$$\begin{aligned}\sum_{i_n=1}^m \|v_{n-1}(z_{i_n})\| &\geq \left|\sum_{i_n=1}^m \langle v_{n-1}(z_{i_n}), x_{i_n}^n\rangle\right| \\ &= \left|\sum_{i_1,\ldots,i_n=1}^m a_{i_1,\ldots,i_n} \sum_{k=1}^m x_{i_1}^1(k)\cdots x_{i_n}^n(k)\right|. \ \square\end{aligned}$$

Corollary 2.3. *For any* $1 \leq p \leq 2$, *the Schatten space with the Schur product* $(S_p, *)$ *is a* Q*-algebra.*

Proof. Davie's criterion (see [7]) tells us that a commutative Banach algebra A is a Q-algebra if and only if there exists a positive constant K such that

$$\left\|\sum_{i_1,\ldots,i_n=1}^m a_{i_1,\ldots,i_n} x_{i_1}\cdots x_{i_n}\right\| \leq K^n \sup_{|t_{i_j}|\leq 1}\left|\sum_{i_1,\ldots,i_n=1}^m a_{i_1\cdots i_n} t_{i_1}\cdots t_{i_n}\right|,$$

for every sequence $x_1,\ldots,x_m \in A$ with $\|x_i\| \leq 1$ and for every choice of $a_{i_1\cdots i_n} \in \mathbf{C}$. Therefore, it is enough to show that

$$\left\|\sum_{i_1,\ldots,i_n=1}^m a_{i_1\cdots i_n} x_{i_1}^1 * \cdots * x_{i_n}^n\right\|_{S_p} \leq K_G^{n-1} \sup_{|t_{i_j}|\leq 1}\left|\sum_{i_1,\ldots,i_n=1}^m a_{i_1\cdots i_n} t_{i_1}\cdots t_{i_n}\right|,$$

for each choice of $a_{i_1\cdots i_n}\in\mathbf{C}$ and $x^1_{i_1},\dots,x^n_{i_n}\in B_{S_p}$.

In fact, Davie's criterion follows for the particular case in which $x^1_i=\cdots=x^n_i$ for every $1\le i\le m$.

So let us choose $a_{i_1\cdots i_n}\in\mathbf{C}$ and $x^1_{i_1},\dots,x^n_{i_n}\in B_{S_p}$. [9, Theorem 4.7(a)] assures that $\|x\|_{S_p}\le\sum_{k,h=1}^\infty\big|x(k,h)\big|$ for each $x=\big(x(k,h)\big)_{k,h\ge1}\in S_p$. Therefore, we have that

$$\begin{aligned}\Big\|\sum_{i_1,\dots,i_n=1}^m a_{i_1\cdots i_n}x^1_{i_1}*\cdots*x^n_{i_n}\Big\|_{S_p}&\le\sum_{k,h=1}^\infty\Big|\sum_{i_1,\dots,i_n=1}^m a_{i_1\cdots i_n}x^1_{i_1}(k,h)\cdots x^n_{i_n}(k,h)\Big|\\&=\sum_{k,h=1}^\infty\varepsilon_{k,h}\sum_{i_1,\dots,i_n=1}^m a_{i_1\cdots i_n}x^1_{i_1}(k,h)\cdots x^n_{i_n}(k,h)\end{aligned}$$

for some $|\varepsilon_{k,h}|=1$. Calling $\bar{x}^1_i(k,h)=\varepsilon_{k,h}x^1_i(k,h)$ (trivially $|\bar{x}^1_i(k,h)|=|x^1_i(k,h)|$ for every k,h) and using Theorem 2.2 for $(a_{i_1\cdots i_n})_{i_j=1}^m$ and $\bar{x}^1_{i_1}$, $x^2_{i_2},\dots,x^n_{i_n}$ we obtain

$$\begin{aligned}&\Big\|\sum_{i_1,\dots,i_n=1}^m a_{i_1\cdots i_n}x^1_{i_1}*\cdots*x^n_{i_n}\Big\|_{S_p}\\&\le K_G^{n-1}\sup_{|t_{i_j}|\le1}\Big|\sum_{i_1,\dots,i_n=1}^m a_{i_1\cdots i_n}t_{i_1}\cdots t_{i_n}\Big|\prod_{j=1}^n\sup_{i_j}\Big(\sum_{k,h=1}^\infty|x^j_{i_j}(k,h)|^2\Big)^{1/2}.\end{aligned}$$

Finally, for each $1\le j\le n$ and each $1\le i_j\le m$, we have that

$$\Big(\sum_{k,h=1}^\infty|x^j_{i_j}(k,h)|^2\Big)^{1/2}=\|x^j_{i_j}\|_{S_2}\le\|x^j_{i_j}\|_{S_p}\le1,$$

and we are done. □

Remark 2.4. The previous corollary proves in fact that $l_p\hat{\otimes}_\pi l_q$ and $l_p(l_q)$ are also Q-algebras with the Schur product when $1\le p,q\le2$.

In the next corollary, in a completely different direction, we point out how Theorem 2.2 can be seen as a result concerning absolutely summing multilinear operators. It improves previous results of Botelho, Meléndez and Tonge in [4] and [15] and it has been recently used by Pellegrino in [17].

Corollary 2.5. *If X_j is an $\mathscr{L}_{\infty,\lambda_j}$-space ($1\le j\le n$), then every multilinear form $T\colon X_1\times\cdots\times X_n\longrightarrow\mathbf{K}$ is $(1;2\dots,2)$-summing and $\|T\|_{(1;2,\dots,2)}\le K_G^{n-1}\prod_{j=1}^n\lambda_j\|T\|$ holds.*

Proof. The proof is quite standard. Because of the local behavior of the $(1;2,\dots,2)$-summing multilinear operators, it is enough to prove that if $T\colon l_\infty^m \times \cdots \times l_\infty^m \longrightarrow \mathbf{K}$ is a multilinear form, then for every $(x_r^j)_{r=1}^m \subset l_\infty^m$ such that $\|(x_r^j)_{r=1}^m\|_2^\omega \le 1$, the following holds

$$\sum_{r=1}^m |T(x_r^1,\dots,x_r^n)| \le K_G^{n-1}\|T\|.$$

Let us define $T_{i_1,\dots,i_n} = T(e_{i_1},\dots,e_{i_n})$ and

$$h_r = \sum_{i_1,\dots,i_n=1}^m T_{i_1\cdots i_n} x_r^1(i_1)\cdots x_r^n(i_n), \quad \theta_r = \begin{cases} \dfrac{|h_r|}{h_r}, & h_r \neq 0,\\ 0, & h_r = 0. \end{cases}$$

If $y_{i_1}^1(r) = \theta_r x_r^1(i_1)$ and $y_{i_j}^j(r) = x_r^j(i_j)$ for $j \ge 2$, we have that

$$\left(\sum_{r=1}^m |y_{i_j}^j(r)|^2\right)^{1/2} \le \left(\sum_{r=1}^m |x_r^j(i_j)|^2\right)^{1/2} \le \|(x_r^j)_{r=1}^m\|_2^\omega \le 1.$$

Therefore $y_{i_j}^j \in B_{l_2^m}$ and so, by Theorem 2.2,

$$\left|\sum_{i_1,\dots,i_n=1}^m T_{i_1\cdots i_n} \sum_{r=1}^m y_{i_1}^1(r)\cdots y_{i_n}^n(r)\right| \le K_G^{n-1}\|T\|.$$

But

$$\begin{aligned}\left|\sum_{i_1,\dots,i_n=1}^m T_{i_1\cdots i_n} \sum_{r=1}^m y_{i_1}^1(r)\cdots y_{i_n}^n(r)\right| &= \left|\sum_{r=1}^m \theta_r \sum_{i_1,\dots,i_n=1}^m T_{i_1\cdots i_n} x_r^1(i_1)\cdots x_r^n(i_n)\right| \\ &= \sum_{r=1}^m |T(x_r^1,\dots,x_r^n)|\end{aligned}$$

and we are done. □

Remark 2.6. In [6] the authors use the fact that every n-linear form $T\colon c_0 \times \cdots \times c_0 \longrightarrow \mathbf{K}$ is $(1;1,\dots,1)$-summing to obtain quite easily certain bounds for polynomials and multilinear forms on c_0, which had been obtained previously and with different techniques in [1], [2] and [25]. The above result could help to obtain better bounds along the same lines.

Finally, we are going to extend Theorem 2.2 to the non-commutative setting. To do that, we are going to use a tricky induction argument of [24] and several deep results of Haagerup, Pisier and Tomczak-Jaegermann. Though it is not the standard in the theory of C^*-algebras, following [21] we will denote $|x|^2 = \frac{1}{2}(xx^* + x^*x)$ when x is an element of a C^*-algebra.

The starting point is the following non-commutative version of Grothendieck's theorem:

Theorem 2.7 ([21, (9.3)]). *There exists a universal constant C such that, if A_1, A_2 are C^*-algebras and $T\colon A_1 \times A_2 \longrightarrow \mathbf{C}$ is a bilinear form, we have that*

$$\sum_{i=1}^{m} \bigl|T(x_i^1, x_i^2)\bigr| \le C\|T\| \left\|\sum_{i=1}^{m} |x_i^1|^2\right\|^{1/2} \left\|\sum_{i=1}^{m} |x_i^2|^2\right\|^{1/2}.$$

Remark 2.8. This result was proved for the first time in [20], in the presence of some approximation condition. The general case is due to Haagerup [11]. The formulation given in [20] and [11] is slightly different, but equivalent (as it is shown in [12, Remark 2.10(b)]).

To generalize it to the n-linear case, we need a pair of results. The first one is a consequence of [21, Theorem 4.1] and [23]. The other can be obtained easily from [21, Theorem 9.4].

Theorem 2.9. *Let A be a C^*-algebra and Y a finite-dimensional Banach space. For every linear operator $u\colon A \longrightarrow Y$ there exists a Hilbert space H and operators $v\colon X \longrightarrow H$ and $w\colon H \longrightarrow Y$ such that $u = wv$ and*

$$\|w\|\,\|v\| \le \bigl(4\sqrt{e}\,C_2(Y)\bigr)^{3/2}\|u\|.$$

Theorem 2.10. *Let A be a C^*-algebra, H a Hilbert space and $u\colon A \longrightarrow H$ a linear operator. Then*

$$\sum_{i=1}^{m} \|u(x_i)\|^2 \le 4\|u\|^2 \left\|\sum_{i=1}^{m} |x_i|^2\right\|,$$

for each finite sequence $(x_i)_{i=1}^m \subset A$.

So let us prove our result:

Theorem 2.11. *If $A_1, \dots, A_n$ are C^*-algebras, every n-linear form $T\colon A_1 \times \cdots \times A_n \longrightarrow \mathbf{C}$ satisfies*

$$\sum_{i=1}^{m} |T(x_i^1, \dots, x_i^n)| \le C^{n-1}\|T\| \left\|\sum_{i=1}^{m} |x_i^1|^2\right\|^{1/2} \cdots \left\|\sum_{i=1}^{m} |x_i^n|^2\right\|^{1/2}$$

where C is an universal constant.

Proof. We are going to reason by induction. The case $n = 2$ is Theorem 2.7. To prove the general case we can suppose $\|T\| \le 1$ and

$$\left\|\sum_{i=1}^{m} |x_i^j|^2\right\|^{1/2} \le 1 \quad \text{for every } 1 \le j \le n.$$

We consider the following operators:

$$\begin{aligned} u&\colon l_2^m \longrightarrow A_1, & u(e_i) &= x_i^1,\\ v&\colon A_1 \longrightarrow \mathscr{L}^{n-1}(A_2,\dots,A_n), & v(x^1) &= T(x^1,\cdot,\dots,\cdot),\\ w&\colon \mathscr{L}^{n-1}(A_2,\dots,A_n) \longrightarrow l_1^m, & w(S) &= \big(S(x_i^2,\dots,x_i^n)\big)_{i=1}^m,\\ i&\colon l_1^m \hookrightarrow l_2^m, & i(e_i) &= e_i. \end{aligned}$$

Clearly $\|v\| \le 1$ and by the induction hypothesis $\|w\| \le C^{n-2}$. Now, by Theorem 2.9 there exists a Hilbert space H and operators $\tilde{v}\colon A_1 \longrightarrow H$, $\tilde{w}\colon H \longrightarrow l_1^m$ such that $\tilde{w}\tilde{v} = wv$ and $\|\tilde{w}\| \le C^{n-2}$, $\|\tilde{v}\| \le C'$, where C' is a universal constant.

We can now consider $\tilde{v}u\colon l_2^m \longrightarrow H$. By Theorem 2.10,

$$\sum_{i=1}^m \|\tilde{v}u(e_i)\|^2 = \sum_{i=1}^m \|\tilde{v}(x_i^1)\| \le 4\|\tilde{v}\|^2 \bigg\|\sum_{i=1}^m |x_i^1|^2\bigg\|.$$

Then, by [9, Corollary 4.8], the Hilbert–Schmidt norm of $\tilde{v}u$ satisfies $\|\tilde{v}u\|_{S_2} \le 4C'$. But, by Grothendieck's theorem and [9, Theorem 4.10], it is also true that the Hilbert–Schmidt norm of $i\tilde{w}\colon H \longrightarrow l_2^m$ satisfies $\|i\tilde{w}\|_{S_2} \le K_G C^{n-2}$.

Therefore, by [19, Theorem 15.5.9], the composition $iwvu\colon l_2^m \longrightarrow l_2^m$ satisfies that $\|iwvu\|_{S_1} \le 4K_G C' C^{n-2}$ and so $\|iwvu\|_{S_1} \le C^{n-1}$ if we choose C greater than $4K_G C'$ and big enough to satisfy the case $n=2$.

Using [19, Theorem 15.5.7], we can conclude that

$$\sum_{i=1}^m |T(x_i^1,\dots,x_i^n)| = \sum_{i=1}^m \big|\big(iwvu(e_i) \mid e_i\big)\big| \le \|iwvu\|_{S_1} \le C^{n-1}$$

where $(\,\cdot\,|\,\cdot\,)$ denotes the scalar product in l_2^m. □

Acknowledgement. This paper was written while the author was visiting Kent State University to which thanks are acknowledged. The author would also like to thank F. Bombal, A. Tonge and I. Villanueva for many helpful conversations.

References

[1] Aron, R., B. Beauzamy, and P. Enflo: Polynomials in many variables: Real vs complex norms. - J. Approx. Theory 74, 1993, 181–198.

[2] Aron, R. M., and J. Globevnik: Analytic functions on c_0. - Rev. Mat. Univ. Complut. Madrid 2, 1989, no. supl., 27–33.

[3] Blei, R. C.: Multidimensional extensions of Grothendieck's inequality and applications. - Ark. Mat. 17, 1979, 51–68.

[4] Botelho, G.: Cotype and absolutely summing multilinear mappings and homogeneous polynomials. - Proc. Royal Irish Acad. 97A, 1997, 145–153.

[5] Carne, T. K.: Banach lattices and extensions of Grothendieck's inequality. - J. London Math. Soc. 21, 1980, 496–516.

[6] Choi, Y. S., S.G. Kim, Y. Meléndez, and A. Tonge: Estimates for absolutely summing norms of polynomials and multilinear maps. - Q. J. Math. 52, 2001, 1–12.

[7] Davie, A. M.: Quotient algebras of uniform algebras. - J. London Math. Soc. 7, 1973, 31–40.

[8] Defant, A., and K. Floret: Tensor Norms and Operator Ideals. - North-Holland, 1993.

[9] Diestel, J., H. Jarchow, and A. Tonge: Absolutely Summing Operators. - Cambridge Univ. Press, 1995.

[10] Grothendieck, A.: Résumé de la théorie métrique des produits tensoriels topologiques. - Bol. Soc. Mat. Sao Paulo 8, 1956, 1–79.

[11] Haagerup, U.: The Grothendieck inequality for bilinear forms on C^*-algebras. - Adv. Math. 56, 1985, 93–116.

[12] Kaijser, S., and A. M. Sinclair: Projective tensor products of C^*-algebras. - Math. Scand. 55, 1984, 161–187.

[13] Kumar, A., and A. M. Sinclair: Equivalence of norms on operator space tensor products of C^*-algebras. - Trans. Amer. Math. Soc. 350, 1998, 2033–2048.

[14] Lindenstrauss, J., and A. Pełczyński: Absolutely summing operators in $\mathscr{L}_p$-spaces and their applications. - Studia Math. 29, 1968, 257–326.

[15] Meléndez, Y., and A. Tonge: Polynomials and the Pietsch domination theorem. - Math. Proc. Roy. Irish Acad. 99A, 1999, 195–212.

[16] Le Merdy, C.: The Schatten space S_4 is a Q-algebra. - Proc. Amer. Math. Soc. 126, 1998, 715–719.

[17] Pelegrino, D.: Cotype and absolutely summing homogeneous polynomials in $\mathscr{L}_p$ spaces. - Studia Math. 157, 2003, 121–131.

[18] Pietsch, A.: Absolut p-summierende Abbildungen in normierten Räumen. - Studia Math. 28, 1967, 333–353.

[19] Pietsch, A.: Operator ideals. - North-Holland, 1980.

[20] Pisier, G.: Grothendieck's theorem for non-commutative C^*-algebras with an appendix on Grothendieck's constants. - J. Funct. Anal. 29, 1978, 397–415.

[21] Pisier, G.: Factorization of Linear Operators and Geometry of Banach Spaces. - CBMS Regional Conference Series 60, American Mathematical Society, 1986.

[22] Pisier, G., and D. Shlyakhtenko: Grothendieck's theorem for operator spaces. - Invent. Math. 150, 2002, 185–217.

[23] Tomczak-Jaegermann, N.: The moduli of smoothness and convexity and the Rademacher averages of the trace classes S_p $(1 \leq p < \infty)$. - Studia Math. 50, 1974, 163–182.

[24] Tonge, A.: The Von Neumann inequality for polynomials in several Hilbert–Schmidt operators. - J. London Math. Soc. 18, 1978, 519–526.

[25] Zalduendo, I.: An estimate for multilinear forms on l^p spaces. - Proc. Royal Irish Acad. 93A, 1993, no. 1, 137–142.

Received 18 January 2005