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RIGIDITY OF COMPLETE MINIMAL HYPERSURFACES IN A HYPERBOLIC SPACE

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Abstract. This paper provides a gap theorem for the first eigenvalue of the stability operator of complete immersed minimal hypersurfaces of dimension no less than five in a hyperbolic space. Namely, we show that an $n(\geq 5)$-dimensional complete immersed minimal hypersurface M in a hyperbolic space is totally geodesic if the first eigenvalue of the stability operator of M is bigger than some concrete constant and if the L^2 norm of the length of the second fundamental form of M grows properly.

1. Introduction

The celebrated Bernstein theorem [2] states that the only complete minimal graphs in $\mathbf{R}^3$ are planes. The works of Fleming [14], De Giorgi [8], Almgren [1] and Simons [22] tell us that the Bernstein Theorem is valid for complete minimal graphs in $\mathbf{R}^{n+1}$ provided that $n \leq 7$. Counterexamples to the theorem for $n \geq 8$ were found by Bombieri–De Giorgi–Giusti [3] and later by Lawson [15]. On the other hand, it has been shown independently by do Carmo–Peng [11], Fischer Colbrie–Schoen [13] that a complete stable minimal surface in $\mathbf{R}^3$ must be a plane. For the higher dimensional case, it is still unknown if a complete oriented stable minimal hypersurface in $\mathbf{R}^{n+1}$ $(3 \leq n \leq 7)$ is a hyperplane. However, do Carmo and Peng have proved the following result.

Theorem A. (do Carmo and Peng [10]) *Let M^n be a complete stable minimal hypersurface in $\mathbf{R}^{n+1}$. If*

$$\lim_{R\to\infty} \frac{\int_{B_p(R)} |A|^2}{R^{2q+2}} = 0, \quad q < \sqrt{\frac{2}{n}},$$

then M is a hyperplane. Here, $B_p(R)$ denotes the geodesic ball of radius R centered at $p \in M$ and A is the second fundamental form of M.

Many interesting generalizations of the above do Carmo–Peng's theorem have been obtained in recent years (cf. [9, 12, 18, 19, 20, 21, 23] etc.). In the present paper, we shall prove similar result for complete minimal hypersurfaces in a hyperbolic space.

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By definition, the hyperbolic space $\mathbf{H}^m$ is a (unique) simply connected complete m-dimensional Riemannian manifold with a constant negative sectional curvature -1.

Before stating our results, we recall some known facts. Let (M, ds^2) be a complete non-compact Riemannian manifold. Let $\mu\colon M \to \mathbf{R}$ be a continuous function and let Δ be Laplacian operator acting on functions of M. We set $L_\mu = \Delta + \mu$ and denote by $\lambda_1(L_\mu, M)$ the first eigenvalue of L_μ. The usual variational characterization of $\lambda_1(L_\mu, M)$ is

$$\lambda_1(L_\mu, M) = \inf_{f\in C_0^\infty(M), f\neq 0} \frac{\int_M (|\nabla f|^2 - \mu f^2)}{\int_M f^2}, \tag{1.1}$$

where $|\nabla f|$ denotes the magnitude of the gradient of f taken with respect to ds^2. When $\mu = 0$, we usually call $\lambda_1(L_0, M)$ the first eigenvalue of M and denote it by $\lambda_1(M)$. It is well known that (cf. [4, 5, 16, 17])

$$\lambda_1(\mathbf{H}^n) = \frac{(n-1)^2}{4}. \tag{1.2}$$

If M is an n-dimensional complete minimal submanifold in $\mathbf{H}^m$, then we have (cf. [7])

$$\lambda_1(M) \geq \frac{(n-1)^2}{4}, \tag{1.3}$$

which is equivalent to say that

$$\int_M |\nabla f|^2 \geq \frac{(n-1)^2}{4} \int_M f^2, \quad \forall f \in C_0^\infty(M). \tag{1.4}$$

If M is a complete minimal hypersurface of $\mathbf{H}^{n+1}$, the stability operator of M is $L_{|A|^2-n}$ and M is said to be stable if $\lambda_1(L_{|A|^2-n}, M) \geq 0$, where A is the second fundamental form of M (cf. [16]). It is easy to see from (1.1) and (1.2) that the first eigenvalue of the the stability operator of a complete totally geodesic hypersurface of $\mathbf{H}^{n+1}$ is $n + \frac{(n-1)^2}{4}$.

In the present paper we prove a gap theorem for the first eigenvalue of the stability operator of complete minimal hypersurfaces in a hyperbolic space. Namely, we have

Theorem 1.1. *Let M be an $n(\geq 2)$-dimensional complete immersed minimal hypersurface in $\mathbf{H}^{n+1}$ and let A be the second fundamental form of M. Suppose that there exists a number $q \in (0, \sqrt{2/n})$ such that*

$$\lim_{R\to\infty} \frac{\int_{B_p(R)} |A|^2}{R^{2q+2}} = 0. \tag{1.5}$$

i) *If $n \geq 6$ and if*

$$\lambda_1(L_{|A|^2-n}, M) > 2n - \frac{(2 - nq^2)(n-1)^2}{4n(1+q)^2}, \tag{1.6}$$

then M is totally geodesic.

ii) *If $n \leq 4$, then*

$$\lambda_1(L_{|A|^2-n}, M) \leq 2n - \frac{(2 - nq^2)n}{2 + 2nq + n}. \tag{1.7}$$

iii) *If* $n = 5,\ q \in (0, 1/5)$ *and if*

$$\lambda_1(L_{|A|^2-5}, M) > 5 + \frac{25(q+1)^2}{10q+7}, \tag{1.8}$$

then M *is totally geodesic.*

iv) *If* $n = 5$ *and if* $q \in [1/5, \sqrt{2/5})$, *then*

$$\lambda_1(L_{|A|^2-5}, M) \leq 5 + \frac{25(q+1)^2}{10q+7}. \tag{1.9}$$

In view of Theorem 1.1, it is interesting to know if a similar result for complete minimal submanifolds in a hyperbolic space holds and to study the following

Problem. What is the sharp lower bound for the first eigenvalue of the stability operator of complete minimal hypersurfaces in a hyperbolic space?

Theorem 1.1 can be generalized to complete hypersurfaces with constant mean curvature in a hyperbolic space. In order to see this, let us recall the following result.

Lemma 1.2. [19] *Let* M *be a complete non-compact immersed submanifold in a Riemannian manifold N. Suppose that* M *has constant mean curvature. If there exist positive constants* $\epsilon,\ a, b$ *and* l, *such that*

$$\int_M |\nabla f|^2 \geq \epsilon \int_M f^2 |A|^a, \quad \forall f \in C_0^\infty(M),$$

and

$$\lim_{R\to+\infty} \frac{\int_{B_R(x)} |A|^b}{R^l} = 0,$$

then M^n *must be minimal.*

Combining Theorem 1.1 and Lemma 1.2, we immediately get

Corollary 1.3. *Let* M *be an* $n(\geq 2)$*-dimensional complete non-compact immersed hypersurface with constant mean curvature in* $\mathbf{H}^{n+1}$ *and let* A *be the second fundamental form of* M. *Assume that there exists a number* $q \in (0, \sqrt{2/n})$ *such that*

$$\lim_{R\to\infty} \frac{\int_{B_p(R)} |A|^2}{R^{2q+2}} = 0. \tag{1.10}$$

i) *If* $n \geq 6$ *and if*

$$\lambda_1(L_{|A|^2-n}, M) > 2n - \frac{(2-nq^2)(n-1)^2}{4n(1+q)^2}, \tag{1.11}$$

then M^n *is totally geodesic.*

ii) *If* $n \leq 4$, *then*

$$\lambda_1(L_{|A|^2-n}, M) \leq 2n - \frac{(2-nq^2)n}{2+2nq+n}. \tag{1.12}$$

iii) *If* $n = 5,\ q \in (0, 1/5)$ *and if*

$$\lambda_1(L_{|A|^2-5}, M) > 5 + \frac{25(q+1)^2}{10q+7}, \tag{1.13}$$

then M *is totally geodesic.*

iv) *If* $n = 5,\ q \in [1/5, \sqrt{2/5})$, *then*

$$\lambda_1(L_{|A|^2-5}, M) \le 5 + \frac{25(q+1)^2}{10q+7}. \tag{1.14}$$

2. A proof of Theorem 1.1

In this section, we will prove the main result in this paper.

Proof of Theorem 1.1. Since M is a minimal hypersurface of $\mathbf{H}^{n+1}$, we have from the Simons' formula that (cf. [6, 22])

$$\frac{1}{2}\Delta|A|^2 = |\nabla A|^2 - |A|^4 - n|A|^2 \tag{2.1}$$

It is well-known that (cf. [24])

$$|\nabla A|^2 - |\nabla|A||^2 \ge \frac{2}{n}|\nabla|A||^2. \tag{2.2}$$

Recalling that $\triangle|A|^2 = 2|A|\triangle|A| + 2|\nabla|A||^2$ and using (2.1) and (2.2) we get the following Kato-type inequality

$$|A|\triangle|A| + |A|^4 + n|A|^2 \ge \frac{2}{n}|\nabla|A||^2. \tag{2.3}$$

Setting

$$\alpha = \lambda_1(L_{|A|^2-n}, M) - n, \tag{2.4}$$

we have from the definition of $\lambda_1(L_{|A|^2-n}, M)$ that

$$\int_M |\nabla f|^2 \ge \int_M |A|^2 f^2 + \alpha\int_M f^2, \quad \forall f \in C_0^\infty(M). \tag{2.5}$$

Setting

$$\gamma = \frac{(n-1)^2}{4}, \tag{2.6}$$

we get from (1.4) that

$$\int_M |\nabla f|^2 \ge \gamma\int_M f^2, \quad \forall f \in C_0^\infty(M). \tag{2.7}$$

Fixing an $x \in [0,1]$, we deduce from (2.5) and (2.7) that

$$x\int_M f^2|A|^2 + (x\alpha + (1-x)\gamma)\int_M f^2 \le \int_M |\nabla f|^2, \quad \forall f \in C_0^\infty(M). \tag{2.8}$$

Plugging $f|A|^{1+q}$ in (2.8) we get

$$\begin{aligned} &x\int_M f^2|A|^{4+2q} + (x\alpha + (1-x)\gamma)\int_M f^2|A|^{2+2q} \le \int_M |\nabla(f|A|^{1+q})|^2 \\ &= (1+q)^2\int_M |A|^{2q}|\nabla|A||^2 f^2 + \int_M |A|^{2q+2}|\nabla f|^2 \\ &\quad + 2(1+q)\int_M |A|^{2q+1} f\langle\nabla f, \nabla|A|\rangle. \end{aligned} \tag{2.9}$$

Multiplying (2.3) by $|A|^{2q}f^2$ and integrating over M, we have

$$\frac{2}{n}\int_M |A|^{2q}f^2|\nabla|A||^2 \le \int_M |A|^{2q+1}f^2\triangle|A| + \int_M |A|^{2q+4}f^2 + n\int_M |A|^{2q+2}f^2. \tag{2.10}$$

It follows from integration by parts that

$$\begin{aligned}\int_M |A|^{2q+1}f^2\triangle|A| &= -\int_M \langle\nabla\left(|A|^{2q+1}f^2\right), \nabla|A|\rangle\\ &= -(2q+1)\int_M |A|^{2q}f^2|\nabla|A||^2 - 2\int_M f|A|^{2q+1}\langle\nabla f, \nabla|A|\rangle.\end{aligned} \tag{2.11}$$

Multiplying (2.10) by $(1+q)$ and using (2.11), one gets

$$\begin{aligned}&(1+q)\left(\frac{2}{n}+2q+1\right)\int_M |A|^{2q}f^2|\nabla|A||^2\\ &\le (1+q)\int_M |A|^{2q+4}f^2 + (q+1)n\int_M |A|^{2q+2}f^2\\ &\quad - 2(1+q)\int_M f|A|^{2q+1}\langle\nabla f, \nabla|A|\rangle.\end{aligned} \tag{2.12}$$

Summing up (2.9) and (2.12) we get

$$\begin{aligned}&x\int_M f^2|A|^{4+2q} + (x\alpha+(1-x)\gamma)\int_M f^2|A|^{2+2q}\\ &+ (1+q)\left(\frac{2}{n}+q\right)\int_m |A|^{2q}f^2|\nabla|A||^2\\ &\le \int_M |A|^{2q+2}|\nabla f|^2 + (1+q)\int_M |A|^{2q+4}f^2 + n(q+1)\int_M |A|^{2+2q}f^2.\end{aligned} \tag{2.13}$$

For any $\epsilon > 0$, we have

$$2\int_M |A|^{2q+1}f\langle\nabla f, \nabla|A|\rangle \le \epsilon\int_M |A|^{2q}|\nabla|A||^2f^2 + \frac{1}{\epsilon}\int_M |A|^{2q+2}|\nabla f|^2. \tag{2.14}$$

Substituting (2.14) into (2.9), we easily obtain

$$\begin{aligned}&x\int_M f^2|A|^{4+2q} + (x\alpha+(1-x)\gamma)\int_M f^2|A|^{2+2q}\\ &\le (1+q)(1+q+\epsilon)\int_M |A|^{2q}|\nabla|A||^2f^2 + \left(1+\frac{1+q}{\epsilon}\right)\int_M |A|^{2q+2}|\nabla f|^2.\end{aligned} \tag{2.15}$$

Multiplying the above inequality by $\frac{\frac{2}{n}+q}{1+q+\epsilon}$ we get

$$\begin{aligned}&\frac{\frac{2}{n}+q}{1+q+\epsilon}\left(x\int_M f^2|A|^{4+2q} + (x\alpha+(1-x)\gamma)\int_M f^2|A|^{2+2q}\right)\\ &\le (1+q)(\frac{2}{n}+q)\int_M |A|^{2q}|\nabla|A||^2f^2 + \frac{(\frac{2}{n}+q)}{\epsilon}\int_M |A|^{2q+2}|\nabla f|^2.\end{aligned} \tag{2.16}$$

Combining (2.13) and (2.16), we have

$$\begin{aligned}&\left(1+\frac{\frac{2}{n}+q}{1+q+\epsilon}\right)\left(x\int_M f^2|A|^{4+2q}+(x\alpha+(1-x)\gamma)\int_M f^2|A|^{2+2q}\right)\\ &\le\left(1+\frac{\frac{2}{n}+q}{\epsilon}\right)\int_M|A|^{2q+2}|\nabla f|^2+(1+q)\int_M|A|^{2q+4}f^2\\ &+n(q+1)\int_M|A|^{2+2q}f^2.\end{aligned}\tag{2.17}$$

Now we consider different cases.

Case i): $n\ge 6$. Setting

$$\beta=n-\frac{(2-nq^2)(n-1)^2}{4n(1+q)^2},\tag{2.18}$$

we know from (1.6) that there exists a constant $\rho>0$ such that

$$\alpha\ge\beta+\rho.\tag{2.19}$$

Since $q\in(0,\sqrt{2/n})$, we can find an $\epsilon>0$ satisfying

$$\frac{(1+q)(1+q+\epsilon)}{\frac{2}{n}+2q+1+\epsilon}+\epsilon<1\tag{2.20}$$

and

$$\rho+\left(\frac{1}{\frac{(1+q)(1+q+\epsilon)}{\frac{2}{n}+2q+1+\epsilon}+\epsilon}-1-\frac{\frac{2}{n}-q^2}{(1+q)^2}\right)\gamma>0.\tag{2.21}$$

Dividing (2.17) by $\left(1+\frac{\frac{2}{n}+q}{1+q+\epsilon}\right)$ and taking

$$x=\frac{(1+q)(1+q+\epsilon)}{\frac{2}{n}+2q+1+\epsilon}+\epsilon,\tag{2.22}$$

one obtains that

$$\epsilon\int_M|A|^{2q+4}f^2+(\gamma+x(\alpha-\gamma)-nx+n\epsilon)\int_M|A|^{2q+2}f^2\le C_1\int_M|A|^{2q+2}|\nabla f|^2,\tag{2.23}$$

for some positive constant C depending only on n,q,ϵ. It follows from (2.18), (2.19), (2.21) and (2.22) that

$$\begin{aligned}\gamma+x(\alpha-\gamma)-nx&=x\left(\left(\frac{1}{x}-1\right)\gamma+\alpha-n\right)\\ &\ge x\left(\left(\frac{1}{\frac{(1+q)(1+q+\epsilon)}{\frac{2}{n}+2q+1+\epsilon}+\epsilon}-1\right)\gamma-\frac{(2-nq^2)(n-1)^2}{4n(1+q)^2}+\rho\right)\\ &=x\left(\rho+\left(\frac{1}{\frac{(1+q)(1+q+\epsilon)}{\frac{2}{n}+2q+1+\epsilon}+\epsilon}-1-\frac{\frac{2}{n}-q^2}{(1+q)^2}\right)\gamma\right)>0.\end{aligned}$$

Thus, we can find an $\epsilon > 0$ and a a positive constant C_1 depending only on n, q, ϵ, ρ, such that

$$\int_M f^2|A|^{2q+4} + \int_M f^2|A|^{2q+2} \le C_1 \int_M |A|^{2q+2}|\nabla f|^2, \ \forall f \in C_0^\infty(M). \tag{2.24}$$

Recall Young's inequality

$$ab \le \frac{\delta^s a^s}{s} + \frac{\delta^{-t}b^t}{t}, \quad \frac{1}{t} + \frac{1}{s} = 1,$$

where $\delta > 0$ is arbitrary and $1 < t, s < +\infty$.

Setting

$$p = \frac{2}{1+q}, \quad s = \frac{q+1}{q}, \quad t = 1+q,$$

then we have

$$pt = 2, \quad s(2q+2-p) = 4+2q, \quad \frac{1}{t} + \frac{1}{s} = 1.$$

It follows from Young's inequality that

$$\begin{aligned} |A|^{2q+2}|\nabla f|^2 &= f^2\left(|A|^{2q+2}\frac{|\nabla f|^2}{f^2}\right) \\ &= f^2\left(|A|^{2q+2-p}|A|^p\frac{|\nabla f|^2}{f^2}\right) \\ &\le f^2\left(\frac{\delta^s}{s}|A|^{s(2q+2-p)} + \frac{\delta^t}{t}\left(|A|^p\frac{|\nabla f|^2}{f^2}\right)^t\right). \end{aligned} \tag{2.25}$$

Putting (2.25) into (2.24) we have

$$\int_M |A|^{2q+4}f^2 \le C_1\frac{q\delta^{\frac{q+1}{q}}}{q+1}\int_M |A|^{2q+4}f^2 + C_1\frac{\delta^{-(1+q)}}{1+q}\int_M |A|^2\frac{|\nabla f|^{2q+2}}{f^{2q+2}},$$

that is,

$$\left(1 - C_1\frac{q\delta^{\frac{q+1}{q}}}{q+1}\right)\int_M |A|^{2q+4}f^2 \le C_1\frac{\delta^{-(1+q)}}{1+q}\int_M |A|^2\frac{|\nabla f|^{2q+2}}{f^{2q+2}}.$$

By choosing δ sufficiently small, we can write the above inequality as

$$\int_M |A|^{2q+4}f^2 \le C_2\int_M |A|^2\frac{|\nabla f|^{2q+2}}{f^{2q+2}}, \tag{2.26}$$

for a new constant $C_2 = C_2(n, \epsilon, q, \rho, \delta)$.

Now, changing in (2.26) f by f^{1+q} we obtain

$$\begin{aligned} \int_M |A|^{2q+4}f^{2q+2} &\le C_2\int_M |A|^2\frac{\left(|\nabla\left(f^{1+q}\right)|^2\right)^{1+q}}{f^{2q(1+q)}} \\ &= C_2(1+q)^{2(1+q)}\int_M |A|^2\frac{f^{2q(q+1)}|\nabla f|^{2q+2}}{f^{2q(q+1)}} \\ &= C_3\int_M |A|^2|\nabla f|^{2q+2}. \end{aligned} \tag{2.27}$$

Fix a $p \in M$ and choose f to be a non-negative cut-off function with the properties

$$|\nabla f| \leq \frac{1}{R}, \quad f = \begin{cases} 1 & \text{on } B_p(R), \\ 0 & \text{on } M \setminus B_p(2R). \end{cases} \tag{2.28}$$

Substituting the above f into (2.27) we get

$$\int_{B_p(R)} |A|^{2q+4} f^{2q+2} \leq \int_M |A|^{2q+4} f^{2q+2} \leq C_3 \int_M |A|^2 |\nabla f|^{2q+2} \leq C_3 \frac{\int_{B_p(2R)} |A|^2}{R^{2q+2}}.$$

Letting $R \to +\infty$ we have, by hypothesis, that the right hand side vanishes. So,

$$\int_M |A|^{2q+2} = 0.$$

This implies $|A| = 0$.

Case ii): $n \leq 4$. Let us assume by contradiction that

$$\alpha > n - \frac{(2 - nq^2)n}{2 + 2nq + n}. \tag{2.29}$$

Taking $x = 1$ in (2.17), we have

$$\begin{aligned} &\left(1 + \frac{\frac{2}{n} + q}{1 + q + \epsilon}\right) \left(\int_M f^2 |A|^{4+2q} + \alpha \int_M f^2 |A|^{2+2q}\right) \\ &\leq \left(1 + \frac{\frac{2}{n} + q}{\epsilon}\right) \int_M |A|^{2q+2} |\nabla f|^2 + (1+q) \int_M |A|^{2q+4} f^2 \\ &\quad + n(q+1) \int_M |A|^{2+2q} f^2. \end{aligned} \tag{2.30}$$

From $q \in (0, \sqrt{2/n})$ and (2.29), we can find an $\epsilon > 0$ satisfying

$$\frac{\frac{2}{n} + q}{1 + q + \epsilon} > q, \quad \left(1 + \frac{\frac{2}{n} + q}{1 + q + \epsilon}\right) \alpha > n(1+q). \tag{2.31}$$

It then follows that there exists an $\epsilon > 0$ such that

$$\int_M f^2 |A|^{2q+4} + \int_M f^2 |A|^{2q+2} \leq C_2 \int_M |A|^{2q+2} |\nabla f|^2, \quad \forall f \in C_0^\infty(M), \tag{2.32}$$

for some positive constant C_2 depending only on n, q and ϵ. Using the same arguments as in the proof of case i) we can conclude that $M = \mathbf{H}^n$ and so $\alpha = \frac{(n-1)^2}{4}$, which contradicts to (2.29) since $n \leq 4$ and $q > 0$.

Cases iii) and iv): Taking $n = 5$ and $x = 1$ in (2.17), we get

$$\begin{aligned} &\left(1 + \frac{\frac{2}{5} + q}{1 + q + \epsilon}\right) \left(\int_M f^2 |A|^{4+2q} + \alpha \int_M f^2 |A|^{2+2q}\right) \\ &\leq \left(1 + \frac{\frac{2}{5} + q}{\epsilon}\right) \int_M |A|^{2q+2} |\nabla f|^2 + (1+q) \int_M |A|^{2q+4} f^2 \\ &\quad + 5(q+1) \int_M |A|^{2+2q} f^2. \end{aligned} \tag{2.33}$$

When $q \in (0, \sqrt{2/5})$ and

$$\alpha > \frac{25(q+1)^2}{10q+7}, \tag{2.34}$$

we can find an $\epsilon > 0$ such that

$$\frac{\frac{2}{5}+q}{1+q+\epsilon} > q, \quad \left(1+\frac{\frac{2}{5}+q}{1+q+\epsilon}\right)\alpha > 5(1+q). \tag{2.35}$$

Thus (2.31) also holds in this case. As in the proof of *case i)*, we know that M is totally geodesic. Therefore $\alpha = 4$, which, combining with (2.34), implies that $q < \frac{1}{5}$. Consequently, we know that items iii) and iv) in Theorem 1.1 hold. □

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