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A NEW APPROACH TO THE CORONA THEOREM FOR DOMAINS BOUNDED BY A $C^{1+\alpha}$ CURVE

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Abstract. We prove the corona theorem for domains whose boundary lies in a $C^{1+\alpha}$ curve. For that, we transfer H^∞ on the complement of the curve onto a Denjoy domain and use the results from Garnett and Jones.

Introduction

Let Γ be an unbounded $C^{1+\alpha}$ curve analytic at ∞, E a compact subset of this curve with positive length and set $\Omega = \mathbf{C}^* \setminus E$. Let us denote the space of bounded analytic functions on Ω by $H^\infty(\Omega)$. The corona theorem for this type of domains was already proved by Moore in [7]. The purpose of this paper is to present a new approach to this result.

Theorem 1. *Let $f_1, f_2, \dots, f_n \in H^\infty(\Omega)$ so that $\delta \leq \max_k |f_k(\omega)| \leq 1$, for all $\omega \in \Omega$ and some $\delta > 0$. Then, there exist $g_1, g_2, \dots, g_n \in H^\infty(\Omega)$ such that $f_1 g_1 + f_2 g_2 + \dots + f_n g_n = 1$ on Ω.*

The functions $\{f_k\}_{k=1}^n$ and $\{g_k\}_{k=1}^n$ are called corona data and corona solutions respectively, and δ and n are the corona constants. When Γ is the real line, the domain Ω is called a Denjoy domain. In this case, the theorem was proved by Garnett and Jones [5].

The first corona problem for simply connected domains was solved by Carleson in 1962 [1]. Since then, the result has been extended to some classes of infinitely connected domains, in particular to domains whose boundary lies in a Lipschitz graph and satisfies a thickness condition [8] or complements of Cantor sets [6].

For our approach, we will apply the following result proved in [2] which allows us to transfer the problem in Ω to a Denjoy domain.

Theorem 2. *Let Γ be an unbounded $C^{1+\alpha}$ curve analytic at ∞, and let ρ denote a conformal map of $\mathbf{R}^2_-$ onto any of the regions bounded by Γ. Then, given a function $g \in L^\infty(\Gamma)$, the Cauchy integral $C_\Gamma(g) \in L^\infty(\mathbf{C})$ if and only if $C_{\mathbf{R}}(f) \in L^\infty(\mathbf{C})$, where f denotes the pullback of g under the conformal mapping ρ.*

This transfer is possible thanks to the existence of a quasiconformal extension of ρ whose complex dilatation, μ, verifies that $|\mu|^2/|y|^{1+\varepsilon}\,dx\,dy$ is a Carleson measure

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relative to **R** for some $\varepsilon = \varepsilon(\alpha) > 0$. In fact, the existence of such an extension characterizes $C^{1+\alpha}$ curves [2, Theorem 1].

The paper is structured as follows: In section 1, we review some definitions and basic facts. The proof of Theorem 1 is presented in section 2.

1. Preliminaries

Let us denote complex variables by $z = x + iy$ and $\omega = \xi + i\eta$. $B_r(z)$ will denote the ball centered at z and radius r and C will represent a positive constant that could be different throughout an inequality. Also, we shall write $\bar\partial = \partial/\partial\bar z = 1/2(\partial_x + i\partial_y)$ and $\partial = \partial/\partial z = 1/2(\partial_x - i\partial_y)$. For a square Q, we will denote by αQ, $\alpha > 0$, the dilation of this square by a scale factor α and by $l(Q)$ its length.

A Jordan curve Γ is said to be of class C^n ($n = 1, 2, \dots$) if it has a parametrization $\varphi(\tau) = f(e^{i\tau}), 0 \le \tau \le 2\pi$, that is n times continuously differentiable and satisfies that $\varphi'(\tau) \ne 0$, $\forall\tau$. Furthermore, it is of class $C^{n+\alpha}$, for $0 < \alpha < 1$, if

$$|\varphi^{(n)}(\tau_1) - \varphi^{(n)}(\tau_2)| \le C|\tau_1 - \tau_2|^\alpha.$$

Given a function F on Γ define its Cauchy integral $f(z) = C_\Gamma(F)(z)$ off Γ by

$$f(z) = \frac{1}{2\pi \mathrm{i}} \int_\Gamma \frac{F(\zeta)}{\zeta - z}\, d\zeta, \quad \zeta \notin \Gamma.$$

We define the jump of $f = C_\Gamma(F)$ across Γ at a point z, $\mathrm{j}(f)(z)$, as $f_+(z) - f_-(z)$, where f_+ and f_- denote the boundary values of f. As the classical Plemelj formula states,

$$f_\pm(z) = \pm\frac{1}{2}F(z) + \frac{1}{2\pi i}\, P.V. \int_\Gamma \frac{F(\omega)}{\omega - z}\, d\omega, \quad z \in \Gamma.$$

Hence $f_+(z) - f_-(z) = F(z)$. Also, f is holomorphic off Γ so that $\bar\partial f = 0$ on $\mathbf{C}\backslash\Gamma$.

A positive measure λ on **C** is called a Carleson measure relative to a given chord-arc curve Γ if there exists a constant $C > 0$ such that $\lambda(B_R(z)) \le CR$ for all $z \in \Gamma$ and $R > 0$. The smallest such C is the norm of λ, $\|\lambda\|_C$. Furthermore, if

$$\lim_{r\to 0} \sup_{R<r} \frac{\lambda(B_z(R))}{R} = 0,$$

then we say that λ is a vanishing Carleson measure or that it satisfies a o(1)-Carleson condition.

2. Proof of the Theorem

Let Ω_+ and Ω_- be the two regions bounded by the $C^{1+\alpha}$ curve Γ and ρ be a conformal map from $\mathbf{R}^2_-$ onto Ω_-. It was proved in [2] that ρ extends to a global quasiconformal map whose dilatation μ satisfies that $\nu = |\mu|^2/|y|^{1+\varepsilon}\, dx\, dy$ is a Carleson measure relative to **R** where $\varepsilon = \varepsilon(\alpha)$. In fact, for this extension, it holds that $|\partial\rho(z)| \simeq |\rho'(\bar z)|$ if $0 < \mathrm{Im}(z) < \varepsilon_0$ for some $\varepsilon_0 = \varepsilon_0(\alpha)$ small enough [2, Proof of Theorem 1].

Besides, since Γ is analytic at ∞, we will assume that μ has compact support. We will keep the notation fixed for the rest of the proof, that is, ρ is a quasiconformal mapping associated to Γ, μ is its complex dilatation and ε is such that ν is a Carleson measure.

Let $E_0 = \rho^{-1}(E) \subset \mathbf{R}$ and $\Omega_0 = \mathbf{C}\backslash E_0$. Note that E_0 is closed and has positive length ([9], Theorem 6.8). Define the space

$$H^\infty(\Omega_0, \mu) = \{f \circ \rho \colon f \in H^\infty(\Omega)\}.$$

Observe that if $g = f \circ \rho \in H^\infty(\Omega_0, \mu)$, then $\bar\partial f = 0$ on Ω translates into $(\bar\partial - \mu\partial)g = 0$ on Ω_0, and as well, the jump of g across E_0 is given by $\mathrm{j}(g) = \mathrm{j}(f) \circ \rho$. Also, as Γ is a $C^{1+\alpha}$ curve, $\lambda = |y||\partial g|^2\,dx\,dy$ is a Carleson measure relative to $\mathbf{R}$ [2, Proof of Theorem 2].

Before proving the corona theorem, we need some preliminary lemmas.

Lemma 2.1. *If $g \in H^\infty(\Omega_0, \mu)$, then $\tau = |\mu||\partial g|\,dx\,dy$ is a vanishing Carleson measure.*

Proof. For any $s \in \mathbf{R}$, $r > 0$:

$$\int_{B_r(s)} \frac{|\mu(z)|^2}{|y|}\,dx\,dy = \int_{B_r(s)} \frac{|\mu(z)|^2}{|y|^{1+\varepsilon}}|y|^\varepsilon\,dx\,dy \lesssim \|\nu\|_C r^{1+\varepsilon}.$$

Therefore,

$$\int_{B_r(s)} |\mu(z)\partial g(z)|\,dx\,dy \le \left(\int_{B_r(s)} \frac{|\mu(z)|^2}{|y|}\,dx\,dy\right)^{1/2} \left(\int_{B_r(s)} |\partial g(z)|^2|y|\,dx\,dy\right)^{1/2}$$

$$\lesssim \|\nu\|_C^{1/2}\|\lambda\|_C^{1/2} r^{1+\varepsilon/2}, \tag{1}$$

and $\tau = |\mu||\partial g|\,dx\,dy$ is a vanishing Carleson measure relative to $\mathbf{R}$. □

Lemma 2.2. *There exists $\varepsilon_0 > 0$ such that if $g \in H^\infty(\Omega_0, \mu)$ and $z \in \Omega_0$ with $0 < |\operatorname{Im}(z)| < \varepsilon_0$, then $|y||\partial g(z)| < C$, where $C = C(\|g\|_\infty, \|\mu\|_\infty)$ and $\varepsilon_0 = \varepsilon_0(\alpha)$.*

Proof. Let $f \in H^\infty(\Omega)$ such that $g = f \circ \rho$. Then, $\delta_\Gamma(\omega)|f'(\omega)| \le C$, $\forall \omega \in \mathbf{C}\backslash\Gamma$ and $C = C(\|f\|_\infty)$.

Let $z \in \mathbf{R}^2_-$ and $\omega = \rho(z)$. Since ρ is conformal on $\mathbf{R}^2_-$, by Koebe's distortion theorem,

$$|y||\partial g(z)| = |y||f'(\rho(z))||\rho'(z)| \simeq \delta_\Gamma(\omega)|f'(\omega)| \le C.$$

If $z \in \mathbf{R}^2_+$, as we mentioned before, we can choose ε_0 so that, if $0 < |\operatorname{Im}(z)| < \varepsilon_0$ then, $|\partial\rho(z)| \simeq |\rho'(\bar z)|$. Hence, as above

$$|y||\partial g(z)| = |y||\partial\rho(z)||f'(\rho(z))| \simeq \delta_\Gamma(\rho(\bar z))|f'(\rho(z))|.$$

By the distortion theorem for quasiconformal mappings $\delta_\Gamma(\rho(\bar z)) \simeq \delta_\Gamma(\rho(z))$ with comparison constants depending on $\|\mu\|_\infty$, which concludes the proof. □

Before stating the next lemma, we will review some facts already developed in [2] which follow Semmes's approach in [10]. Let $g \in H^\infty(\Omega_0, \mu)$, then $g = f \circ \rho$ for some $f \in H^\infty(\Omega)$. Consider now the jump of g, $\mathrm{j}(g)$, and set $\tilde g = C_{\mathbf{R}}(\mathrm{j}(g))$. If we define $G = g - \tilde g$, then $\bar\partial G = \mu\partial g$ on Ω_0 and since G has no jump across E_0, we can consider that this equation holds on all $\mathbf{C}$ in the sense of distributions. We can then apply Cauchy's formula to obtain

$$G(z_0) = \frac{1}{\pi \mathrm{i}} \int_{\mathbf{C}} \frac{\bar\partial G(z)}{z - z_0}\,dx\,dy = \frac{1}{\pi \mathrm{i}} \int_{\mathbf{C}} \frac{\mu(z)\partial g(z)}{z - z_0}\,dx\,dy, \quad \text{for all } z_0 \in \mathbf{C}.$$

Lemma 2.3. *Assume that $\operatorname{supp}(\mu) \subset Q$ for some Q centered at a real point with length $R \le \varepsilon_0/4$. Let $g \in H^\infty(\Omega_0, \mu)$ and $\tilde g \in H^\infty(\Omega_0)$ so that $\mathrm{j}(g) = \mathrm{j}(\tilde g)$ and set $G = g - \tilde g$. Then for all $z \in \mathbf{C}$, $|G(z)| \le CR^{\varepsilon/(2+\varepsilon)}$, where $C = C(\|g\|_\infty, \|\nu\|_C)$.*

Proof. Consider $z_0 = x_0 + \mathrm{i}y_0 \in (2Q\backslash \mathbf{R})$. Since $\bar\partial G = \mu\partial g$ and $\operatorname{supp}(\mu) \subset Q$, then

$$|G(z_0)| \lesssim \int_{\mathbf{C}} \frac{|\mu(z)\partial g(z)|}{|z-z_0|}\,dx\,dy = \int_Q \frac{|\mu(z)\partial g(z)|}{|z-z_0|}\,dx\,dy$$
$$(2)\qquad = \int_{Q_0} \frac{|\mu(z)\partial g(z)|}{|z-z_0|}\,dx\,dy + \int_{Q\backslash Q_0} \frac{|\mu(z)\partial g(z)|}{|z-z_0|}\,dx\,dy,$$

where Q_0 is the square centered at z_0 and length $l(Q_0) = |y_0|$. To bound the first integral in (2), set $p = 2+\varepsilon$ and $q = (2+\varepsilon)/(1+\varepsilon)$. Then

$$(3)\quad \int_{Q_0} \frac{|\mu(z)\partial g(z)|}{|z-z_0|}\,dx\,dy \le \left(\int_{Q_0} |\mu(z)\partial g(z)|^{2+\varepsilon} dx\,dy\right)^{\frac{1}{2+\varepsilon}} \left(\int_{Q_0} |z-z_0|^{-\frac{2+\varepsilon}{1+\varepsilon}} dx\,dy\right)^{\frac{1+\varepsilon}{2+\varepsilon}}.$$

As $\nu = |\mu|^2/|y|^{1+\varepsilon}$ is a Carleson measure relative to $\mathbf{R}$ and $|y| \ge |y_0|/2$ for $z \in Q_0$, we obtain by lemma 2.2:

$$\int_{Q_0} |\mu(z)\partial g(z)|^{2+\varepsilon}\,dx\,dy \lesssim \int_{Q_0} |\mu(z)|^{2+\varepsilon} \frac{1}{|y|^{2+\varepsilon}}\,dx\,dy$$
$$(4)\qquad \lesssim \frac{2}{|y_0|} \int_{2Q_0} \frac{|\mu(z)|^2}{|y|^{1+\varepsilon}}\,dx\,dy \le 4\|\nu\|_C.$$

Let us now consider $B_0 = B_r(z_0)$ so that $r \simeq |y_0|$ and $Q_0 \subset B_0$. By changing variables to polar coordinates,

$$(5)\quad \int_{Q_0} |z-z_0|^{-\frac{2+\varepsilon}{1+\varepsilon}}\,dx\,dy \le \int_{B_0} |z-z_0|^{-\frac{2+\varepsilon}{1+\varepsilon}}\,dx\,dy \le C(\varepsilon) r^{\frac{\varepsilon}{1+\varepsilon}} \simeq C(\varepsilon)|y_0|^{\frac{\varepsilon}{1+\varepsilon}}.$$

Therefore, by (3), (4) and (5)

$$(6)\qquad \int_{Q_0} \frac{|\mu(z)\partial g(z)|}{|z-z_0|}\,dx\,dy \lesssim C(\|\nu\|_C,\varepsilon)|y_0|^{\frac{\varepsilon}{2+\varepsilon}} \lesssim C(\|\nu\|_C,\varepsilon) R^{\frac{\varepsilon}{2+\varepsilon}}.$$

To bound the second integral in (2), consider an open cover of $Q\backslash Q_0$ with squares, Q_i, centered at z_0 and length $l(Q_i) = 2^i|y_0|$, $i \ge 1$. Note that it is sufficient a cover with M squares such that $M \lesssim \log_2(R/|y_0|)$. Then, by (1)

$$\int_{Q\backslash Q_0} \frac{|\mu(z)\partial g(z)|}{|z-z_0|}\,dx\,dy \lesssim \sum_{i=1}^{M} \frac{2^{-i}}{|y_0|} \int_{Q_i\backslash Q_{i-1}} |\mu(z)\partial g(z)|\,dx\,dy$$
$$(7)\qquad \lesssim \sum_{i=1}^{M} \frac{2^{-i}}{|y_0|} (2^i|y_0|)^{1+\varepsilon/2} \lesssim |y_0|^{\varepsilon/2}(2^{\varepsilon/2})^M \lesssim R^{\varepsilon/2}.$$

Therefore, by (2), (6) and (7), $|G(z_0)| \le C(\|\nu\|_C, \|\lambda\|_C, \varepsilon) R^{\varepsilon/(2+\varepsilon)}$.

For $z_0 \in (2Q \bigcap \mathbf{R})$, let Q^i be the square centered at z_0 and length $l(Q^i) = 2^{2-i}R$, $i \ge 0$. Since $\bar\partial G = \mu\partial g$ and $\operatorname{supp}(\mu) \subset Q$, by (1)

$$|G(z_0)| \lesssim \int_Q \frac{|\mu(z)\partial g(z)|}{|z-z_0|}\,dx\,dy = \sum_{i\ge 0} \int_{Q^i\backslash Q^{i+1}} \frac{|\mu(z)\partial g(z)|}{|z-z_0|}\,dx\,dy$$
$$\lesssim \frac{1}{R}\sum_{i\ge 0} 2^i \int_{Q^i} |\mu(z)\partial g(z)|\,dx\,dy \lesssim \frac{1}{R}\sum_{i\ge 0} 2^i l(Q^i)^{1+\varepsilon/2} \lesssim R^{\varepsilon/2}.$$

Therefore, $|G(z_0)| \le CR^{\varepsilon}/2$ for $C = C(\|\mu\|_C, \|\lambda\|_C, \varepsilon)$.

Finally, let $z_0 \in \mathbf{C}\backslash 2Q$. Then, by (1)

$$|G(z_0)| \lesssim \int_Q \frac{|\mu(z)\partial g(z)|}{|z-z_0|}\,dx\,dy \le \frac{1}{R}\int_Q |\mu(z)\partial g(z)|\,dx\,dy$$
$$\le C(\|\nu\|_C, \|\lambda\|_C)R^{\varepsilon/2}. \qquad \square$$

We now prove Theorem 1:

Theorem 1. *Let $f_1, f_2, \dots, f_n \in H^\infty(\Omega)$ so that $\delta \le \max_j |f_j(\omega)| \le 1$, for all $\omega \in \Omega$ and some $\delta > 0$. Then, there exist $g_1, g_2, \dots, g_n \in H^\infty(\Omega)$ such that $f_1g_1 + f_2g_2 + \dots + f_ng_n = 1$ on Ω.*

Proof. Gamelin [3] showed that it is sufficient to prove it locally, that is, that for $\zeta \in \Gamma$ there exists a neighborhood of ζ on which it is true and such that the size of this neighborhood is determined by δ, n and other parameters concerning Γ (see also [4, p. 358]).

We can then assume that $\mu(z) = 0$ outside a square Q centered at a real point with length R, for a small enough $R = R(n, \delta, \Gamma)$ to be determined later. To see this, consider the solution $\tilde\rho$ of the Beltrami equation $\bar\partial\tilde\rho = \mu\partial\tilde\rho$ for $z \in Q$, $\bar\partial\tilde\rho = 0$ otherwise. Then, $\rho = F \circ \tilde\rho$ where F is an univalent function in the region $\tilde\rho(Q)$, and therefore it will be enough to prove the corona theorem for the domain $\tilde\Omega = \mathbf{C}\backslash\tilde\rho(E_0)$.

Since the dilatation coefficient $\tilde\mu = \mu\chi_Q$ obviously satisfies that $|\tilde\mu|^2/|y|^{1+\varepsilon}\,dx\,dy$ is a Carleson measure, we know that $\tilde\Gamma = \tilde\rho(\mathbf{R})$ is also a $C^{1+\tilde\alpha}$ curve for $\tilde\alpha = \tilde\alpha(\alpha, \|\mu\|_\infty)$ ([2], Theorem 1) and therefore all the previous lemmas apply if we replace Γ, μ and ρ by the corresponding $\tilde\Gamma, \tilde\mu$ and $\tilde\rho$. To avoid excessive use of notation, we will drop the tilde notation.

Let $f_k^* = f_k \circ \rho$ on Ω_0. Then, the jump of f_k^* across E_0 is indeed the pullback of $\mathrm{j}(f_k)$ under the mapping ρ, that is, $\mathrm{j}(f_k^*) = \mathrm{j}(f_k) \circ \rho$. Note that $f_1^*, \dots, f_n^* \in H^\infty(\Omega_0, \mu)$.

Set $\tilde f_k = C_{\mathbf{R}}(\mathrm{j}(f_k^*))$. By Theorem 2, $\tilde f_k \in H^\infty(\Omega_0)$. First, we want to show that $\tilde f_k$ are corona data in Ω_0. So, let $G_k = f_k^* - \tilde f_k$ and $z_0 \in \Omega_0$. Then, there exists $1 \le j \le n$ such that $\delta \le |f_j^*(z_0)| \le |G_j(z_0)| + |\tilde f_j(z_0)|$. By lemma 2.3, $|G_j(z_0)| \lesssim R^{\varepsilon/(2+\varepsilon)} \le \delta/2$ for a sufficiently small R and therefore $\delta/2 \le |\tilde f_j(z_0)|$.

According to Garnett and Jones' theorem for Denjoy domains [5], there exist $\tilde h_1, \tilde h_2, \dots, \tilde h_n \in H^\infty(\Omega_0)$ such that $\tilde f_1\tilde h_1 + \dots \tilde f_n\tilde h_n = 1$ with $\|\tilde h_k\|_\infty \le C(n, \delta)$.

Define $H_k^* = \mathrm{j}(\tilde h_k)$. Then, $H_k^* \in L^\infty(\mathbf{R})$ and $\tilde h_k = C_{\mathbf{R}}(H_k^*)$. Set $H_k = H_k^* \circ \rho^{-1}$ on Γ and define $h_k = C_\Gamma(H_k)$. Although $\{h_k\}_{k=1}^n \subset H^\infty(\Omega)$ by Theorem 2, they are not corona solutions as they do not verify that $f_1h_1 + f_2h_2 + \dots f_nh_n = 1$ on Ω.

Consider the analytic functions $g_k(\omega) = h_k(\omega)/(\sum f_j(\omega)h_j(\omega))$, $1 \le k \le n$, on Ω. They clearly satisfy that $\sum g_jf_j = 1$. We just need to prove that $g_1, g_2, \dots g_n$ are also bounded. For that, it is sufficient to show that $\sum f_kh_k$ is close to 1.

Let us denote $h_k^* = h_k \circ \rho \in H^\infty(\Omega_0, \mu)$. Note that $\mathrm{j}(h_k^*) = \mathrm{j}(\tilde h_k)$. For any $z \in \Omega_0$ and by lemma 2.3:

$$|\sum_{k=1}^n f_k(\rho(z))h_k(\rho(z)) - 1| = |\sum_{k=1}^n f_k^*(z)h_k^*(z) - \sum_{i=1}^n \tilde f_k(z)\tilde h_k(z)|$$
$$\le \sum_{k=1}^n |f_k^*(z)||h_k^*(z) - \tilde h_k(z)| + \sum_{k=1}^n |\tilde h_k(z)||\tilde f_k(z) - f_k^*(z)|$$

$$\lesssim nR^{\varepsilon/(2+\varepsilon)} + nC(n,\delta)R^{\varepsilon/(2+\varepsilon)} \leq 1/2 \tag{8}$$

for a sufficiently small R. □

As a final remark, this new approach encourages us to find solutions to the corona problem for domains bounded by other quasicircles. For that, one would need to find conditions on μ so that we can transfer H^∞ on the complement of a curve onto the corresponding Denjoy domain.

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