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CORRIGENDUM TO "ATOMIC DECOMPOSITION OF HARDY–MORREY SPACES WITH VARIABLE EXPONENTS"

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Abstract. We correct a technical error in [2, Theorem 5.1].

In this note, we correct a technical error appeared in the proof of [2, Theorem 5.1]. At the end of p. 49 of [2], we use the inequalities

$$\begin{aligned} c|\varphi * g^j(x)| &\le (\mathcal{M}g^j)(x) \\ &\le (\mathcal{M}f)(x)\chi_{\{x\in\mathbf{R}^n\colon (\mathcal{M}f)(x)\le 2^j\}}(x) + 2^j \sum_{k\in\mathbf{N}} \frac{l(Q_k^j)^{n+d+1}}{(l(Q_k^j)+|x-x_k^j|)^{n+d+1}} \\ &\le C2^j \end{aligned}$$

to prove that $g^j \to 0$ in $\mathcal{S}'(\mathbf{R}^n)$ as $j \to -\infty$. This is an error as the last inequality does not necessarily hold.

Most importantly, the result $\lim_{j\to-\infty} g^j = 0$ in $\mathcal{S}'(\mathbf{R}^n)$ is valid. We now give a proof of the result $\lim_{j\to-\infty} g^j = 0$ in $\mathcal{S}'(\mathbf{R}^n)$ by using the ideas in [2, p. 50]. The reader is referred to [2] for the notions used in this note.

For any $Q \in \mathbf{B}$, [2, Propoisition 5.4] yields

$$\begin{aligned} \int_Q |(\mathcal{M}g^j)(x)|\,dx &\le C2^j \int_Q dx + C2^j \int_Q \sum_{k\in\mathbf{N}} \frac{l(Q_k^j)^{n+d_{p(\cdot)}+1}}{(l(Q_k^j)+|x-x_k^j|)^{n+d_{p(\cdot)}+1}}\,dx \\ &\le C2^j|Q| + C2^j \sum_{k\in\mathbf{N}} \int_{\mathbf{R}^n} \chi_Q(x)((\mathrm{M}\,\chi_{Q_k^j})(x))^{(n+d_{p(\cdot)}+1)/n}\,dx. \end{aligned}$$

By using [1, Chapter II, Theorem 2.12], we obtain

$$\begin{aligned} \int_{\mathbf{R}^n} ((\mathrm{M}\,\chi_{Q_k^j})(x))^{(n+d_{p(\cdot)}+1)/n}\chi_Q(x)\,dx &\le \int_{\mathbf{R}^n} (\chi_{Q_k^j}(x))^{(n+d_{p(\cdot)}+1)/n}(\mathrm{M}\,\chi_Q)(x)\,dx \\ &= \int_{\mathbf{R}^n} \chi_{Q_k^j}(x)(\mathrm{M}\,\chi_Q)(x)\,dx \\ &= \int_{Q_k^j} (\mathrm{M}\,\chi_Q)(x)\,dx \end{aligned}$$

because $(n + d_{p(\cdot)} + 1)/n > 1$.

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Therefore, the finite intersection property of $\{Q_k^j\}$ yields

$$\int_Q |(\mathcal{M} g^j)(x)|\,dx \le C2^j|Q| + C2^j \sum_{k\in\mathbf{N}} \int_{Q_k^j} (\mathrm{M}\,\chi_Q)(x)\,dx$$
$$\le C2^j|Q| + C2^j \int_{O^j} (\mathrm{M}\,\chi_Q)(x)\,dx.$$

Consequently, for any $\varphi \in \mathcal{S}(\mathbf{R}^n)$ and $x \in \mathbf{R}^n$, we have

$$|g^j * \varphi(x)| \le C\frac{1}{|B(x,1)|}\int_{B(x,1)} |M_1^*(g^j,\varphi)(y)|\,dy$$
$$\le C\int_{B(x,1)} |(\mathcal{M} g^j)(y)|\,dy$$
$$\le C2^j|B(x,1)| + C2^j\int_{O^j} (\mathrm{M}\,\chi_{B(x,1)})(y)\,dx$$

for some $C>0$.

Thus, it suffices to show that $2^j\int_{O^j}(\mathrm{M}\,\chi_{B(x,1)})(y)\,dx \to 0$ as $j\to-\infty$.

Let $0<r<\min(1,m_{p(\cdot)})$ and $B^k = B(x,2^k)\backslash B(x,2^{k-1})$ when $k\ge 1$ and $B^0 = B(x,1)$. We find that

$$\int_{O^j}(\mathrm{M}\,\chi_{B(x,1)})(y)\,dx \le \int_{O^j}(1+|x-y|)^{-n}\,dy \le C\sum_{k=0}^{\infty} 2^{-kn}\int_{O^j}\chi_{B^k}(y)\,dy$$
$$\le C\sum_{k=0}^{\infty}\frac{1}{|B(x,2^k)|}\|\chi_{O^j\cap B(x,2^k)}\|_{L^{p(\cdot)/r}(\mathbf{R}^n)}\|\chi_{B(x,2^k)}\|_{L^{(p(\cdot)/r)'}(\mathbf{R}^n)}$$
$$\le C\sum_{k=0}^{\infty}\frac{u(x,2^k)^r}{\|\chi_{B(x,2^k)}\|_{L^{p(\cdot)/r}(\mathbf{R}^n)}}\|\chi_{O^j}\|^r_{\mathcal{M}_{p(\cdot),u}}.$$

As $u^r \in \mathcal{W}_{h_{p(\cdot)/r}}$, [2, Lemma 3.3] yields

$$2^j\int_{O^j}(\mathrm{M}\,\chi_{Q(x,1)})(y)\,dx \le 2^jC\|\chi_{O^j}\|^r_{\mathcal{M}_{p(\cdot),u}}$$

for some $C>0$ independent of $j\in\mathbf{Z}$.

In view of $0<r<1$ and $\|\chi_{O^j}\|^r_{\mathcal{M}_{p(\cdot),u}} \le 2^{-jr}\|\mathcal{M}f\|^r_{\mathcal{M}_{p(\cdot),u}} = 2^{-jr}\|f\|^r_{\mathcal{H}_{p(\cdot),u}}$, we have

$$\lim_{j\to-\infty}2^j\int_{O^j}(\mathrm{M}\,\chi_{Q(x,1)})(y)\,dx \le C\lim_{j\to-\infty}2^{j-jr}\|f\|^r_{\mathcal{H}_{p(\cdot),u}} = 0.$$

References

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